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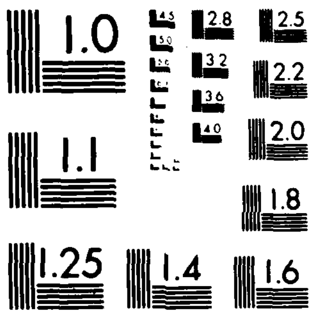
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AFIT/GAE/AA/83D-1

MINIMUM TIME TURNS WITH

DIRECT SIDEFORCE

THESIS

AFIT/GAE/AA/83D-1 Michael R. Brinson

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MINIMUM TIME TURNS WITH DIRECT SIDEFORCE

THESIS

Presented to the Faculty of the School of Engineering

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by

Michael R. Brinson, B.S.

Graduate Aeronautical Engineering

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Table of Contents

List of Figures

List of Symbols

x - distance (x-direction)

y - distance (y-direction)

h - altitude

V - velocity

γ - flight path angle

χ - heading angle

S - wing area

W - weight

T - thrust

L - lift

D - drag

Q - sideforce

ε - thrust angle of attack

ζ - thrust side slip angle

μ - bank angle

α - angle of attach

α - thrust control variable

Σ - sideforce control variable

g_o - gravitational acceleration at sea level

ρ - density

ρ_σ - density at sea level

σ - density ratio

τ - nondimensional time

C_L - lift coefficient

C_D - drag coefficient

C_{L_α} - lift curve slope

C_{D_o} - parasite drag coefficient

K_1 - induced drag due to lift parameter

K_2 - induced drag due to sideforce parameter

G - performance index (t_f)

F - augmented performance index

$\underline{X}$ - state vector

U - control vector

M - final condition vector

H - Variational Hamiltonian

λ - Lagrange multiplier vector (differential equations)

ν - Lagrange multiplier vector (final conditions)

A - unknown parameter vector

$(\ )_i$ - initial value

$(\ )_f$ - final value

$(\ )_c$ - corner value

$(\ )_A$ - (A arbitrary)

$\dot{a}$ - $\frac{da}{dt}$ (a arbitrary)

A^T - a transposed (a arbitrary)

Abstract

The objective of the study is to find the optimal trajectories and corresponding minimum turning times for a high performance aircraft with and without direct sideforce to perform a prescribed turn. These trajectories and times are then compared to evaluate the benefit of direct sideforce. Optimal control theory is applied to solve the minimum time to turn optimal control problem using a suboptimal control problem approach and a second order parameter optimization method.

The results indicate that the use of direct sideforce is beneficial in reducing turning time. In addition, the use of sideforce causes a small loss of energy for initial velocities lower and much higher than the corner velocity.

MINIMUM TIME TURNS WITH DIRECT SIDEFORCE

I Introduction

Background

In air-to-air combat, the aircraft with a higher turn rate capability has an advantage. The turn rate of a conventionally controlled aircraft is completely determined by specifying the load factor and velocity. The maximum turn rate occurs at maximum load factor and one particular airspeed. The maximum load factor, however, is bounded by two limits, a maximum structural limit and a maximum lift limit. Since these limits are heavily influenced by other design requirements, aircraft designed for similar roles tend to have similar maximum turn rates.

One possible way of inproving turning time without sacrificing performance in other design areas may be through the use of conventional control such as direct sideforce control. The Air Force is currently flight testing the Advanced Fighter Technology Integrator (AFTI) aircraft which has the capability of generating both sideforce and lift without the accompaning yawing and pitching moments required by conventionally controlled aircraft.

Another important parameter in air-to-air engagements is specific energy. Since maximum performance turns generally cause a loss of energy, the presence of sideforce may minimize the loss or even allow a gain of energy.

Several studies such as Humphreys, Hennig, Bolding and Helgeson (4) and Well and Berger (7) have been published on the optimal trajectories and control schedules of conventional aircraft necessary to minimize turning time. However, none have considered the use of a translational force such as direct sideforce. This study, therefore, considers the effects on the optimum turning maneuver of applying direct sideforce.

Problem Statement

The problem is to find the control schedule which will minimize the time to maneuver a high performance aircraft from a set of initial conditions to a set of prescribed final conditions.

The controls to be optimized are bank angle, thrust, angle of attack, and sideforce. Additionally, the controls must not cause the aircraft to exceed its capability or design limits. These limits include maximum angle of attack or buffet limit, maximum structural load factor, minimum and maximum thrust, and the maximum allowable sideforce.

The objective of this study is to find the optimal controls, subject to constraints, which will minimize the turning time both with and without direct sideforce. From this study, the benefit of sideforce will then be evaluated.

Scope

The scope of this study is to find the trajectories which minimizes turning time for a variety of initial conditions. Additionally, the scope involves determining the effects of sideforce application on turning time.

Assumptions

The assumptions made in this study can be divided into two parts, those used in modeling the aircraft and those made in deriving the equations of motion. First, the aircraft is modeled as a point mass with three degrees of freedom. This simplification is common for the type of trajectories considered in this study (4:91, 7:84). The fuel burn during the maneuver is considered negligible due to the short time it takes to complete a turn. Another simplification is that the error from the thrust vector not being colinear with the aircraft centerline is negligible. The coefficient of lift was assumed to be a linear function of angle of attack up to the stall limit. The drag coefficient was modeled as a function of the square of the lift coefficient and a linear function of sideforce. Standard incompressible aerodynamic theory shows these assumptions to be valid (8:11.8) The choice for a linear sideforce term in the drag equation is substantiated by windtunnel data from a similarly configured experimental Air Force aircraft. The maximum thrust was considered constant in this study. Since the maneuver is carried out within a narrow altitude band, the air density, and hence thrust, remains nearly constant.

The equations of motion were developed (1) assuming a flat earth with a constant gravitational acceleration. All atmospheric data were obtained from the 1962 Standard Atmosphere defined by NASA (2). In all cases the turning maneuver was initiated from straight and level flight, but the control variables angle of attack, band angle, sideforce, and thrust were allowed to vary instantaneously. This simplification was made to reduce the problem to manageable

complexity and allow the effects of different control schedules to be evaluated without contamination from a arbitrarily assumed generic control response data.

Approach

To determine the effect of sideforce on turning flight, a realistic aircraft model must be derived and the parameters of the maneuver determined. The particular aircraft model and two turning situations were taken from Johnson's study (5:7-8). This was done in order to check the optimization computer code written by Johnson (5) after modification to include sideforce. From these two baseline cases, without sideforce, various parameters of the aircraft and maneuver were varied. These parameters include initial airspeed, thrust to weight ratio, and drag due to lift coefficient. The aircraft parameters were changed from the basic model in an attempt to compare the results with the qualitative conclusions of Well and Berger (7).

The following sections will define the turning problem, summarize the optimal and suboptimal control approaches, and discuss solutions and results.

Summary of Current Knowledge

Henning, Bolding, and Helgeson (4) generated several minimum turning time trajectories for a generic aircraft without sideforce. Results were presented for two initial velocities, several maximum thrust to weight ratios, and several final conditions. For the study, only the cases with a final heading of 180 degrees and a zero final flight path angle were useful for comparison results.

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Johnson (5) and Peterson (6) use the results from two cases in (4) to verify a suboptimal numerical technique for finding the optimal control schedules required to minimize the turning time.

Well and Berger (7) used a different optimization technique to obtain general trends in optimal controls. One parameter of interest is the inclination of the plane in which the turn is performed. Well and Berger (7) show that for any initial velocity below a specific value, the optimum maneuver is a vertical Split-S. For initial velocities faster than some definable value, a half loop manuever results in minimum time to turn. The optimum turning plane varies between these two extremes for initial velocities between the two key velocities. Unfortunately, Well and Berger (7) did not provide the specific aircraft characteristics used to generate their results. Although direct comparison of results cannot be made, the general trends can be used to qualitatively check the results beyond the two cases used by Johnson (5) and Peterson (6).

II The Minimum Time Turn Problem

In order to evaluate the benefits of direct sideforce in a turning maneuver, it is first necessary to define a baseline from which a difference in performance due to the sideforce can be shown.

Establishing the baseline involves defining the turning maneuver as well as developing the governing equations of motion and a realistic model of an aircraft.

The Maneuver

The initial conditions for all cases in this study are straight and level flight at 13,990 feet. From these conditions and a specified initial velocity, the aircraft turns to a final heading of 180° with a final flight path angle of zero.

Equations of Motion

The equations of motion for flight of a point mass aircraft over a flat earth are derived by Miele (1:42-49) as:

$$\dot{X} = V\cos\gamma\cos\chi \tag{1}$$

$$\dot{Y} = V\cos\gamma\sin\chi \tag{2}$$

$$\dot{h} = V\sin\gamma \tag{3}$$

$$T\cos\varepsilon\cos\zeta - D - mg\sin\gamma - m\dot{V} = 0 \tag{4}$$

$$T\cos\varepsilon\sin\zeta - Q + mg\sin\mu\cos\gamma + mV(-\dot{\chi}\cos\mu\cos\gamma + \dot{\gamma}\sin\mu) = 0 \tag{5}$$

$$T\sin\varepsilon + L - mg\cos\mu\cos\gamma - mV(\dot{\chi}\sin\mu\cos\gamma + \dot{\gamma}\cos\mu) = 0 \tag{6}$$

Assuming the error due to neglecting the thrust sideslip angle is small, then $\sin\zeta = 0$, $\cos\zeta = 1$, and $\varepsilon = \gamma$. Substituting these values into equations (1) through (6) and rearranging yields:

$$\dot{X} = V\cos\gamma\cos\chi \tag{7}$$

$$\dot{Y} = V\cos\gamma\sin\chi \tag{8}$$

$$\dot{h} = V\sin\gamma \tag{9}$$

$$\dot{V} = g((T/W)\cos\gamma - D/W - \sin\gamma) \tag{10}$$

$$\dot{\chi} = \frac{g}{V\cos\gamma} \quad ((T/W)\sin\alpha\sin\mu - (Q/W)\cos\mu + (L/W)\sin\mu) \tag{11}$$

$$\dot{\gamma} = \frac{g}{V} \quad ((T/W)\sin\alpha\cos\mu + (Q/W)\sin\mu + (L/W)\cos\mu - \cos\gamma) \tag{12}$$

These equations are written in the wind axes and model the motion of an aircraft with respect to an earth fixed coordinate frame. The state variables are X, Y, h, V, χ, and γ. The control variables are α, μ, T, and Q. New forms of the control variables T and Q will be defined later. The aerodynamic forces of L and D are discussed in the next section. The two variables, g and W, are assumed to be constant during the maneuver.

Aircraft Forces

The aerodynamic forces of lift and drag can be expressed as follows:

$$L = \frac{\rho_o \sigma V^2 S C_L}{2} \tag{13}$$

$$D = \frac{\rho_o \sigma V^2 S C_D}{2} \tag{14}$$

Expressions for C_L and C_D can be assumed from their airfoil theory for conventional aircraft. However, the sideforce in this study is assumed to be generated by aerodynamic surfaces. Therefore, the aircraft drag must include the effects of generating sideforce.

An expression for this effect was obtained from windtunnel data of an experimental Air Force aircraft with similar characteristics to the model used in this study. The expressions for C_L and C_D thus derived are:

$$C_L = C_{L_\alpha} \alpha \tag{15}$$

$$C_D = C_{D_o} + K_1 C_L^2 + K_2 Q \tag{16}$$

Substituting these equations into (13) and (14) and dividing by the weight for use in the equations of motion yield:

$$\frac{L}{W} = \frac{\rho_o \sigma V^2 S}{2W} \quad C_{L_\alpha} \alpha \tag{17}$$

$$\frac{D}{W} = \frac{\rho_o \sigma V^2 S}{2W} \quad (C_{D_o} + K_1 C_L^2 + K_2 Q) \tag{18}$$

Finally, the thrust to weight ratio and the sideforce to weight ratio are needed for use in the equations of motion. The maximum thrust and the maximum sideforce are both assumed to be constant throughout the turn. Therefore, thrust and sideforce can be defined as:

$$T = T_{MAX} \, \pi \tag{19}$$

$$Q = Q_{MAX} \, \Sigma \tag{20}$$

Now π and Σ become the control variables for thrust and sideforce, respectively. Forming thrust and sideforce to weight ratios yields,

$$T/W = \frac{T_{MAX}}{W} \pi = \left(\frac{T}{W}\right)_{MAX} \pi \tag{21}$$

$$Q/W = \frac{Q_{MAX}}{W} \Sigma = \left(\frac{Q}{W}\right)_{MAX} \Sigma \tag{22}$$

The actual values of the aircraft parameters used in this study are listed below. In some cases the values for $(T/W)_{MAX}$ and K_1 were varied to show their influence on the problem. These special cases are detailed in Chapter VI so only the nominal values are listed here.

$W = 12{,}150$ lb $\qquad\qquad$ $S = 237$ sq.ft.

$C_{L_\alpha} = 5.0$ $\alpha_{MAX} = .2$ radians

$C_{D_o} = .02$ $(L/W)_{MAX} = 7.22$

Note that the induced drag factor, K_1, can be calculated for a given aircraft geometry. Typical values range from 0.1 up to 0.3 for fighter type aircraft (8:11.9, E.7). A value of 0.05, therefore, indicates this is a very low drag aircraft.

Atmosphere

A standard atmosphere defined in (2) was used throughout the study. The density ratio is expressed as:

$$\sigma = \frac{\rho}{\rho_o} = [1 - (\frac{n-1}{n}) \frac{g_o}{RT_o} h]^{\frac{1}{n-1}} \tag{23}$$

where,

$\rho_o = .002377$ slugs/ft^3

$g_o = 32.174$ ft/sec^2

$T_o = 518.688°R$

$n = 1.235$

$R = 1715$ ft^2/sec^2-°R

The Control Variable Constraints

There are aircraft related constraints on all control variables except bank angle. These constraints are the maximum lift limit, maximum load factor, minimum and maximum thrust and maximum sideforce.

The angle of attack is limited by either the maximum lift limit or the maximum load factor, which ever is less. This effect on maximum angle of attack is shown as function of airspeed in Fig 1. The velocity where these two limits meet is called the corner

velocity and is discussed in section V.

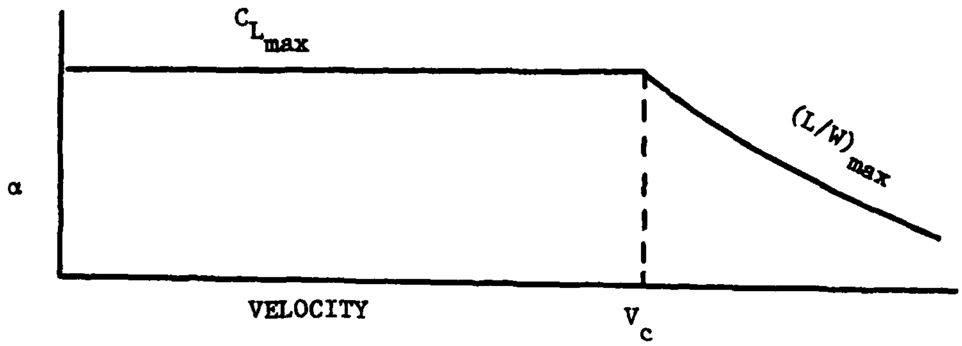

Fig 1. Maximum Angle of attack vs velocity

The thrust is bounded on the lower side by the minimum thrust which is defined as zero in this study. From this and equation (21), the thrust control variable is limited to,

$$0 \le \pi \le 1 \tag{24}$$

Sideforce can be generated in either direction up to the maximum value in equation (22). Therefore, the sideforce control must fall in the range,

$$-1 \le \Sigma \le 1 \tag{25}$$

These limits and their effects on the problem are discussed more fully in Section V.

III The Optimal Control Problem

The optimal control approach is discussed in this chapter to define the basic conditions to be satisfied by the optimal solution. The discussion also shows the complexity and the inherent solution difficulties of the approach.

Optimal Control Problem

The optimal control problem requires finding the functional ralationships for the control variables that will minimize the performance index,

$$G = t_f \tag{26}$$

subject to the differential constraints expressed vectorially as

$$\underline{\dot{X}} = f \tag{27}$$

where X is the state vector and f is the vector containing the right side of Eqs (7) to (12). The problem is also subject to the control constraints defined in the previous chapter and to the final conditions expressed as:

$$M_1 = \gamma_f \tag{28}$$

$$M_2 = \chi_f - 180° \tag{29}$$

Thus, the final conditions are satisfied when,

$$M = \binom{M_1}{M_2} = 0 \tag{30}$$

The Calculus of Variations is applied to find the necessary and sufficient conditions to be satisfied by the optimal control variables.

Conditions to be Satisfied

From Johnson's work (5:13-14) the augmented performance index, J, is defined by:

$$J = G + \nu^T M + \int_{t_i}^{t_f} \lambda^T (f - \dot{\underline{X}}) dt \tag{31}$$

where ν and λ are Lagrange multiplier vectors, then the variational Hamiltonian, H, is expressed as

$$H = \lambda^T f \tag{32}$$

The optimal controls, expressed vectorially as U, must satisfy the first variational requirement or the Euler-Lagrange equations:

$$\dot{\lambda} = -H_X{}^T = -\left(\frac{\partial H}{\partial X}\right)^T \tag{33}$$

$$\dot{X} = H_\lambda{}^T = \left(\frac{\partial H}{\partial \lambda}\right)^T \tag{34}$$

$$U_{opt} = \min_U (H) \tag{35}$$

Equation (35) states that the optimal controls are those which minimize H. The optimal solution must also satisfy the transversality conditions and corner conditions:

$$H_i = G_{t_i} = 0 \tag{36}$$

$$H_f = G_{t_f} = 0 \tag{37}$$

$$\lambda_i{}^T + G_{\underline{x}_i} = 0 \tag{38}$$

$$\lambda_f{}^T - G_{\underline{x}_f} = 0 \tag{39}$$

$$\Delta\ (H) - G_{t_c} = 0 \tag{40}$$

$$\Delta\ (\lambda^T) + G_{x_c} = 0 \tag{41}$$

$$G_\nu = 0 \tag{42}$$

Explicitly solving the optimal control problem is at best complex. If the controls intersect boundaries or are singular, the difficulty increases. It is desirable therefore, to find a

technique which will yield a reasonable solution. The Suboptimal Control Method developed by Hull and Edgeman (3) fulfills these requirements.

IV The Suboptimal Control Problem

The suboptimal control approach used in this study transforms the optimal control problem into a parameters optimization problem. This is accomplished by assuming a known mathmatical form with a number of unknown constants for the control variables. This technique is used by several numerical optimization methods including the well known Raleigh-Ritz metho and reduces the problem to one of finding the coefficients which satisfy the conditions of the problem.

If the unknown constants of the controls are formed into a vector B, the controls can be written as

$$U = U(t,B) \tag{43}$$

And, the vector of all unknowns in the problem can be represented as

$$A = [t_f, B]^T \tag{44}$$

For any vector A, the equations of motion can be integrated from $t=0$ to $t=t_f$ to yield a set of final state variables expressed functionally as

$$\underline{X}_f = \underline{X}_f(A) \tag{45}$$

Therefore, the performance index, G, as well as the final conditions vector, M, are functions of the A vector only. The problem is then to find the A vector which minimizes the turning time,

$$G = G(A) \tag{46}$$

subject to the equations of motion

$$\dot{\underline{X}} = f(\underline{X}, A, t) \tag{47}$$

the physical control constraints and the final conditions,

$$M(A) = 0 \tag{48}$$

<u>Conditions to be Satisfied</u>

An augmented performance, F, can be defined as:

$$F(A, \nu) = G(A) + \nu^T M(A) \tag{49}$$

Ordinary differential calculas requires two conditions to be satisfied:

$$F_A{}^T(A, \nu) = G_A + \nu^T M_A = 0 \tag{50}$$

$$F\nu^T(A, \nu) = M(A) = 0 \tag{51}$$

where M_A and G_A are defined as

$$M_A = \frac{\partial M}{\partial A} \tag{52}$$

$$G_A = \frac{\partial G}{\partial A} \tag{53}$$

The gradient, $F_A{}^T$, is a column vector containing first order information or the slope indicating the change in F with respect to changes in each element of A. $F_A{}^T$ then reveals the direction which A must change to drive F to zero. Since equations (50) and (51) are the only two conditions imposed by the suboptimal control approach, it is clearly much less complex than the optimal approach and requires less knowledge of the peculiarities of each specific problem.

<u>Method of Solution</u>

Hull and Edgeman (3) present a second order parameter optimization technique and an algorithm ideally suited to be used in a digital computer program. Basically the paper developes two vector equations which yield the changes necessary in the

Lagrange multipliers, ν, and the A vector to drive F_A^T and M to zero. These equations are:

$$\delta\nu = (M_A F_{AA}^{-1} M_A^T)^{-1} (-PM_A F_{AA}^{-1} F_A^T + QM) \quad (54)$$

$$\delta A = -F_{AA}^{-1} (PF_A^T + M_A^T \delta\nu) \quad (55)$$

where

$$M_A = \frac{\partial M}{\partial A} \quad (56)$$

$$F_{AA} = \frac{\partial^2 F}{\partial A^2} \quad (57)$$

and P and Q are scaling factors which control optimization and satisfaction of the end conditions, respectively. The algorithm to solve the suboptimal problem was written into computer code by Johnson (5) and presented here as follows:

1. guess A and ν;
2. integrate the equations of motion to obtain $\underline{X}_f$;
3. compute M, M_A, M_{AA}, F_A, and F_{AA};
4. select values for P and Q and compute $\delta\nu$ and δA;
5. set $A = A + \delta A$ and $\nu = \nu + \delta\nu$;
6. check convergence criteria and if unsatisfied go to step 2.

Guessing an initial A vector is relatively simple since the elements of A are physical parameters of the problem. However, finding an initial value for the ν vector and picking P and Q are not obvious. Hull and Edgeman (3) developed a first order-approach which eliminates the need to guess ν. The vector equations for this purpose are:

$$\nu = (M_A M_A^T)^{-1} [(Q/P)M - M_A G_A^T] \quad (58)$$

$$\delta A = -PF_A^T \tag{59}$$

The first order information of Equations (58) and (59) can be used to iterate to some region close to a solution. Then a switch can be made to the second order method (Eqs (54) and (55)) for quicker final convergence.

In this study the value of Q remained fixed at (1) throughout the iteration. The optimization weighting factor was initially set small and allowed to increase until the norm $||\delta A||$ began to get too large. If δA becomes too large, the iteration process may skip over the solution and increase convergence time or prevent convergence completely. When a solution is eminent, P is set equal to one and norms $||F_A^T||$ and $||M||$ are monitored. When they are less than some small positive number, convergence is achieved.

V Solving the Minimum Turning Time Problem

The solution of the suboptimal control problem is sufficiently complex to require the use of numerical techniques. In order to implement these techniques, the optimal problem defined in an earlier chapter must be adapted. In this chapter the equations of motion are presented in their final form, the control variables are discussed further, and the numerical methods used to implement the algorithm in Chapter VI are specified.

Equations of Motion

One step of the suboptimal control algorithm requires the equation of motion to be integrated from $t = 0$ to $t = t_f$. However, the final time is an unknown in the problem. For convenience, the time can be nondimensionalized as

$$\tau = \frac{t}{t_f} \quad (0 \leq \tau \leq 1) \tag{60}$$

Then the time dependence in the equations of motion can be eliminated by transforming the equations to functions of the non-dimensional time, τ. By the use of the chain rule,

$$\dot{X} = \frac{dX}{dt} = \frac{dX}{d\tau}\,\frac{d\tau}{dt} = \frac{dX}{d\tau}\left(\frac{1}{t_f}\right) \tag{61}$$

or

$$\frac{dX}{d\tau} = t_f\dot{X} \tag{62}$$

The other variables in Eqs (7) through (12) can similarly be transformed. Doing this and substituting the expressions for aerodynamic forces, atmospheric parameters, and control variables the equations can be rewritten as:

$$\frac{dX}{d\tau} = t_f V \cos\gamma \cos\chi \tag{63}$$

$$\frac{dY}{d\tau} = t_f V\cos\gamma\sin\chi \tag{64}$$

$$\frac{dH}{d\tau} = t_f V\sin\gamma \tag{65}$$

$$\frac{dV}{d\tau} = t_f\{48.261\pi\cos\alpha - [1.4917\text{x}10^{-5} + 9.3238\text{x}10^{-4}\alpha^2$$

$$+ 6.9547\text{x}10^{-8}|\Sigma|][1-6.8823\text{x}10^{-6}h]^{4.2553}V^2$$

$$- 32.174\sin\gamma\} \tag{66}$$

$$\frac{dX}{d\tau} = \frac{t_f}{V\cos\gamma}[48.261\pi\ \sin\alpha\sin\mu - 16.087\Sigma\cos\mu$$

$$+ 3.7295\text{x}10^{-3}(1-6.8823\text{x}10^{-6}h)^{4.2553}V^2\alpha\sin\mu] \tag{67}$$

$$\frac{d\gamma}{d\tau} = \frac{t_f}{V}[48.261\pi\ \sin\alpha\cos\mu + 16.087\Sigma\sin\mu$$

$$+ 3.7295\text{x}10^{-3}(1-6.8823\text{x}10^{-6}h)^{4.2553}V^2\alpha\cos\mu$$

$$- 32.174\cos\gamma] \tag{68}$$

The Control Variables

Equations (24) and (25) give the ranges of the thrust and sideforce controls respectively. The angle of attack is limited by either one of two constraints. For speeds below the corner velocity, it is bounded by the maximum lift limit given by

$$\alpha \leq 0.2 \text{ radians} \tag{69}$$

For speeds above the corner velocity, the angle of attack is limited by the maximum load factor.

$$(\frac{L}{W}) \leq 7.22 \tag{70}$$

Substituting the relationship from Equation (17) yields

$$\frac{\rho_o\sigma V^2 SC_L}{2W} \leq 7.22 \tag{71}$$

Further substitution of the known parameters simplifies Equation (71) to

$$\alpha \leq \frac{62286.8}{\sigma\, V^2} \tag{72}$$

Thus, Equations (69) and (72) form the complete boundary for angle of attack as shown in Fig 1. The corner velocity is defined as the velocity at which the lift limit equals the load factor limit and can be calculated by equating Equations (69) and (72).

$$0.2 = \frac{62286.8}{\sigma V_C^{\,2}} \tag{73}$$

or

$$V_C = 558.06\sigma^{-\frac{1}{2}} \tag{74}$$

The corner velocity plays an important part in a minimum time turning maneuver and is discussed further in Chapter VI.

The constraints on thrust, sideforce, and angle of attack are brought into the problem when the optimization algorithm causes the controls to exceed their limits. When this occurs, the control is set equal to its limit. In addition to limiting the control variable itself, the effect on the optimization routine must be eliminated if a convergence is to be attained. This is done by checking the sign of the F_A terms for each limit bound control. A negative F_A term indicates the control must be increased to drive F_A to zero. But, if the control is already at its maximum limit, F_A cannot be driven to zero. For these cases the effected control is no longer a variable in the problem because its value is fixed at the maximum or minimum values. Therefore, the matrices used to calculate $\delta \underline{A}$ and $\delta \underline{\nu}$ are computed without the presence of that control.

Numerical Forms for the Control Variables

In this study, all control variables are described by Chebyshev polynomials with τ as the independent variable. The polynomials, T_i for $i = 1$ to $i = 4$ are

$$T_1 = 1 \tag{75}$$

$$T_2 = 2\tau - 1 \tag{76}$$

$$T_3 = 8\tau^2 - 8\tau + 1 \tag{77}$$

$$T_4 = 32\tau^3 - 48\tau^2 + 18\tau - 1 \tag{78}$$

The control variables are then described by

$$\mu = \sum_{k=1}^{NM\upsilon} B_k T_k \tag{79}$$

$$\pi = \sum_{\ell=1}^{NPI} C_\ell T_\ell \tag{80}$$

$$\alpha = \sum_{m=1}^{NA} D_m T_m \tag{81}$$

$$\Sigma = \sum_{n=1}^{NS} E_n T_n \tag{82}$$

where B, C, D, and E are the unknown coefficients and $NM\upsilon$, NPI, NA, and NS are the number of coefficients for μ, π, α, and Σ respectively. It should be mentioned that Johnson's results (5) show little changes to either the control time histories or the turning times for controls descirbed by more than four coefficients. For this reason the control equations in this study were limited to third order. The actual values and number of coefficients representing each control is discussed in Chapter VI.

Numerical Methods

The numerical algorithm used in the suboptimal control approach requires the matrices M, M_A, M_{AA}, F_A, and F_{AA} to be computed. From Ref 3:484, F_A and F_{AA} can be calculated from

$$F_A = G_A + \nu^T M_A \tag{83}$$

$$F_{AA} = G_{AA} + \nu_1 M_{AA1} + \nu_2 M_{AA2} \tag{84}$$

Since $G = t_f$, G_A and G_{AA} can be analytically determined, M, M_A, and M_{AA} are the only unknowns.

To evaluate M, Equations (63) through (68) can be integrated from $\tau = 0$ to $\tau = 1$. The M_A and M_{AA} are computed by a central difference numerical derivative technique. This technique requires the individual elements of A to be perturbed both positively and negatively one by one. After each perturbation the equations of motion must be integrated from $\tau = 0$ to $\tau = 1$. The M_A is calculated by:

$$M_{A_n} = \frac{M_+ - M_-}{2\delta n} \tag{85}$$

where M_{A_n} are the two M_A elements calculated by perturbing the nth element of A. M_+ and M_- are the final conditions computed by changing A_n a small amount, δn. The second derivative is computed similarly by:

$$M_{A_n A_m} = \frac{M_+ - 2M_+ M_-}{\delta_n^{\,2}} \tag{86}$$

when $n = m$ and,

$$M_{A_n A_m} = \frac{M_{++} - M_{+-} - M_{-+} + M_{--}}{4\delta n \delta m} \tag{87}$$

if $n \neq m$. The two subscripts indicate two elements. A_n and A_m must be perturbed both positively and negatively. The equations of motion then must be integrated for each combination of A_n and A_m although the M_{AA} matrices are symmetric. The total number of times the equations of motion must be integrated for one iteration of the algorithm is then given by:

$$N = \tfrac{1}{2}(n^2+n+3) \tag{88}$$

where n is the number of elements in the A vector and can be calculated as

$$n = 1 + NNV + NPI + NA + NS \tag{89}$$

From Equation (88) it is apparent an efficient numerical integration routine is imperative. A fifth order, Runge-Kutta integration routine with variable step size is used in this study. Another problem which influences the integration routine is the discontinous slope of the angle of attack limit at the corner velocity.

If the velocity traverses the corner velocity as the equations of motion are integrated, small inaccuracies occur due to the angle of attack limit changing across the corner velocity. These inaccuracies, while not severe when compared to the absolute values of the state variables, are large enough to prevent the optimization algorithm from converging to an answer. Therefore, the step size must be controlled as velocity approaches the corner velocity. But, since the variable of integration is nondimensional time, τ, an iterative procedure is required to find the τ at which velocity equals the corner velocity. Once the corner velocity is passed the integration routine can be allowed to optimize step size to minimize computation time.

VI Results

The results of this study are divided into two parts. The first part discusses the effects of sideforce on the minimum turning time problem. The second part shows the results of attempting to match the qualitative data presented by Well and Berger (7).

In all cases, two of the controls stayed consistantly against their limits. The angle of attack always migrated to its limit of 0.2 and Σ always went to the maximum negative limit of -1.0 when that control was included in the problem. For this reason, these two controls were represented by constant coefficients only instead of higher order polynomials. This greatly reduced the number of control coefficients and sped up convergence time. One other variable, thrust, was limited to a single coefficient. Though Johnson's results (4:61) concluded a near bang-bang type control for thrust yields smaller turning times, real world engines have finite spool up and spool down times. Therefore, the short duration of the maneuver would not realistically allow large excursions in thrust. In addition, it was felt the computation time could more constructively be spent investigating the effect of sideforce if thrust remained constant throughout the turn. The plot in Figure 2 shows the optimal value for the thrust coefficient as a function of initial velocity for the case where all controls are represented by single coefficients. Values for thrust are also shown for variations in aircraft parameters such as T/W and the drag due to lift coefficient, K_1. These parameter variations are discussed later in this chapter.

The Effects of Sideforce

The effect of sideforce on turning time is shown in Figure 3. The graph shows that sideforce has more influence on the final time the farther the initial velocity is from the corner velocity. It also shows that even if the turn for a conventional aircraft is initiated at the corner velocity, which is the speed that yields the quickest turn, the use of sideforce reduces the turning time though not very much. The reason for this can be seen in Figure 4. The presence of a direct sideforce generates a sideslip angle. This sideslip angle then reduces the angle through which the velocity vector must be turned by conventional means. Hence, the turning time is reduced. One would assume that more sideforce would generate more sideslip and thereby reduce the turning time even more. Figure 5 illustrates the effect of various values of maximum sideforce up to one "g" lateral acceleration. The changing slope of these curves indicates that even greater benefit can be realized if the sideforce is allowed to increase above the one "g" limit shown in Figures 6 through 8. If the specific energy is given as

$$E_s = (h + \frac{V^2}{2g}) \tag{90}$$

where $g = 32.174$ ft/sec^2, then the change is specific energy during the maneuver can be calculated

$$E_s = E_{s_f} - E_{s_i} = (h_f - h_i) + \frac{(V_f^{\,2} - V_i^{\,2})}{2g} \tag{91}$$

where subscripts i and f denote initial and final conditions respectively. Values of ΔE_s are plotted as a function of initial airspeed both with and without sideforce in Figure 9. As can be seen at the low initial speeds, an aircraft using sideforce gains less energy during

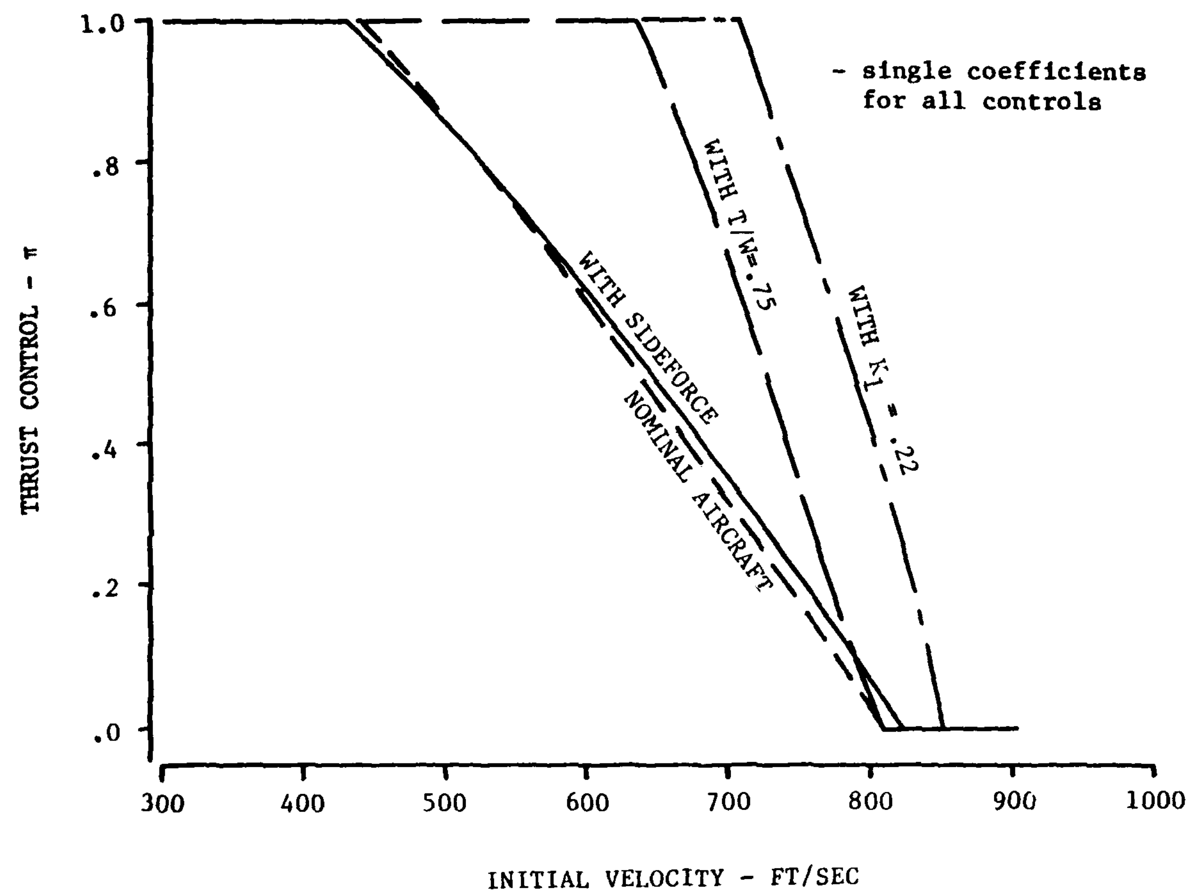

Fig. 2. Optimum Thrust Coeffient vs. Initial Velocity

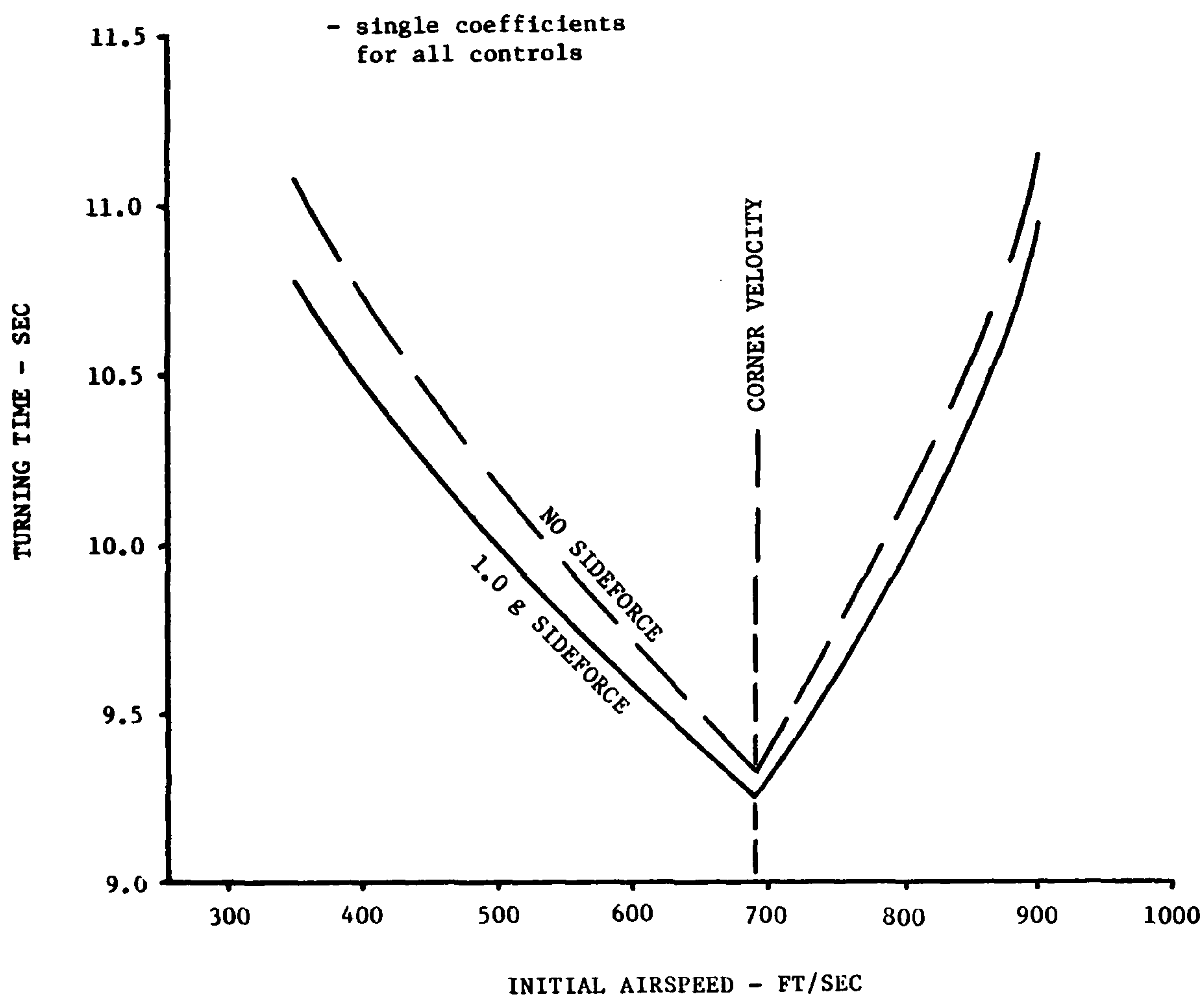

Fig. 3. Turning Time vs. Initial Velocity

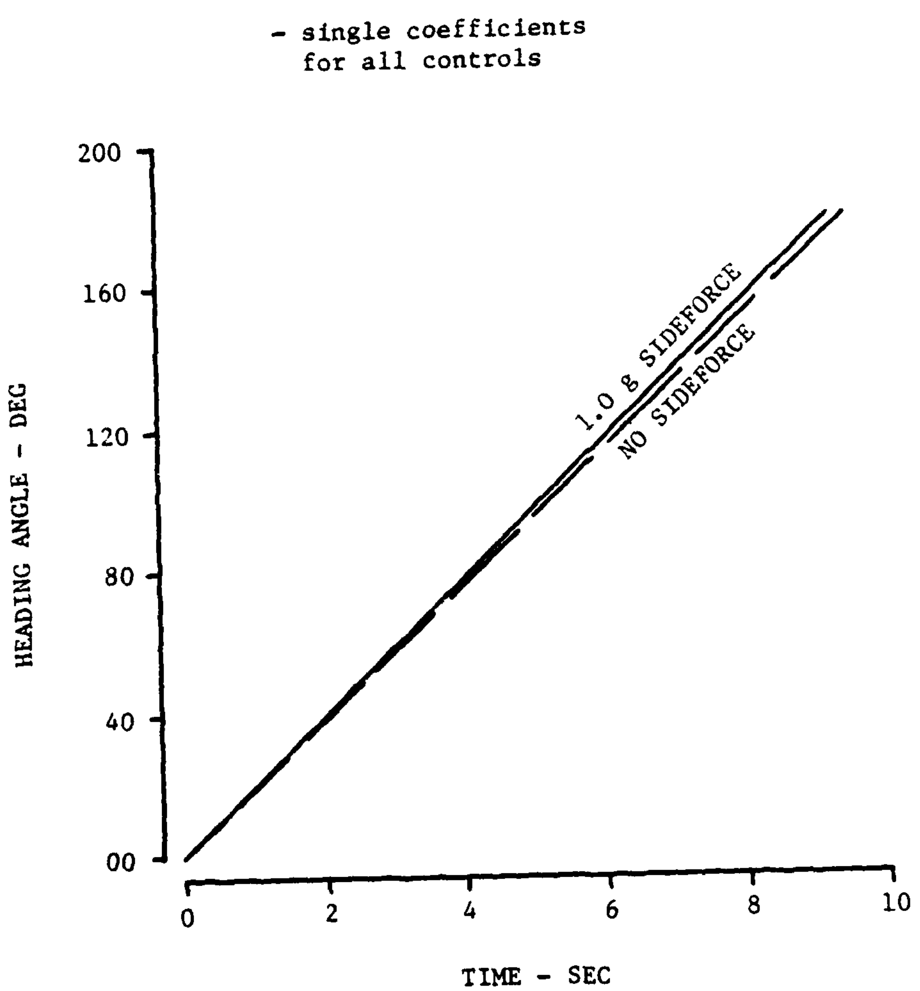

Fig. 4. Heading Angle Time History

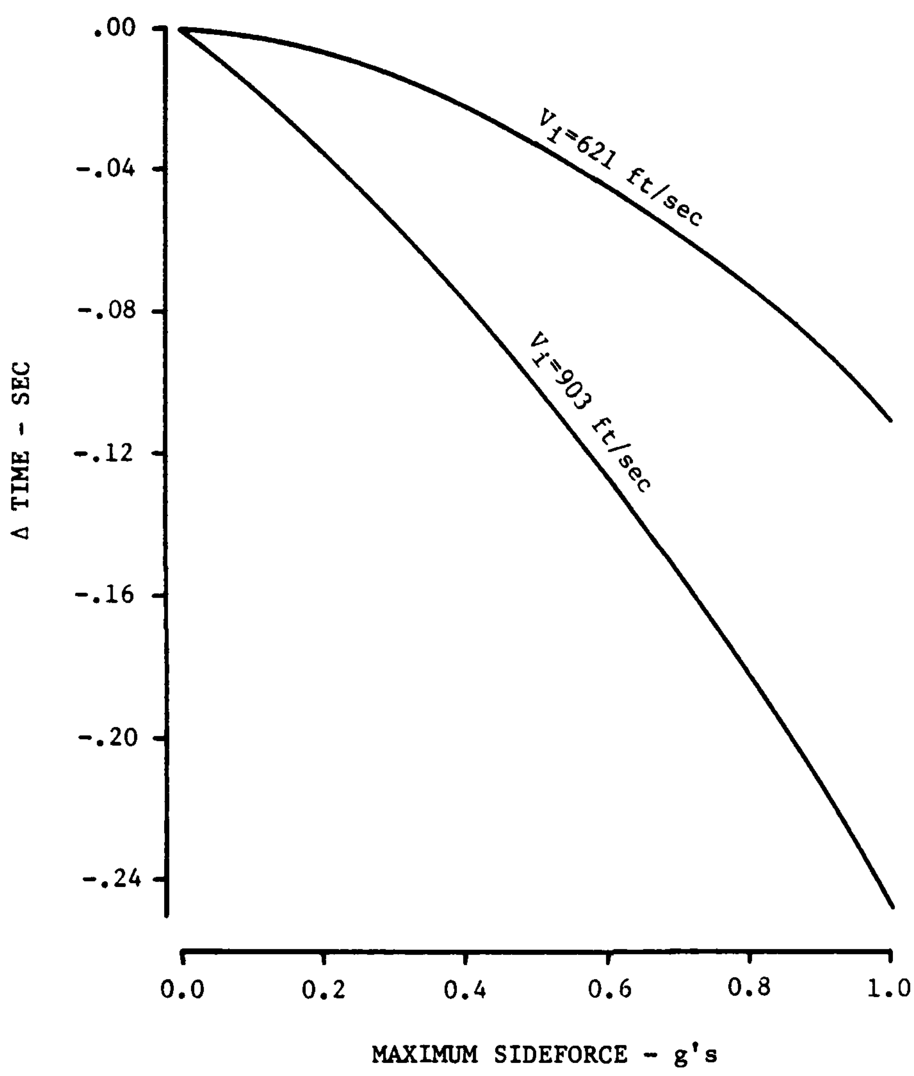

Fig. 5. Change in Turning Time vs. Maximum Sideforce

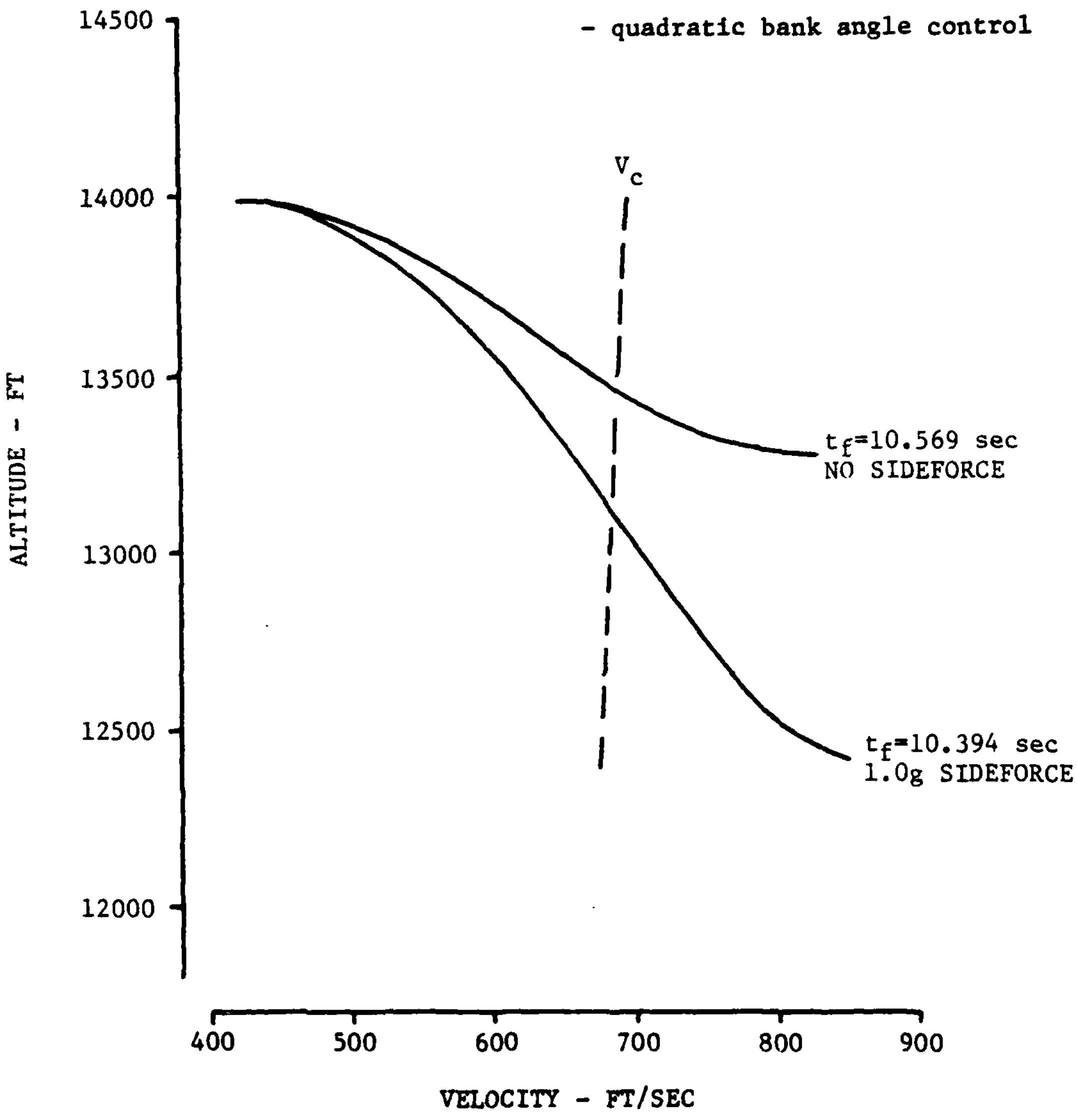

Fig. 6. Altitude vs. Velocity Trajectory for V_i = 420 ft/sec

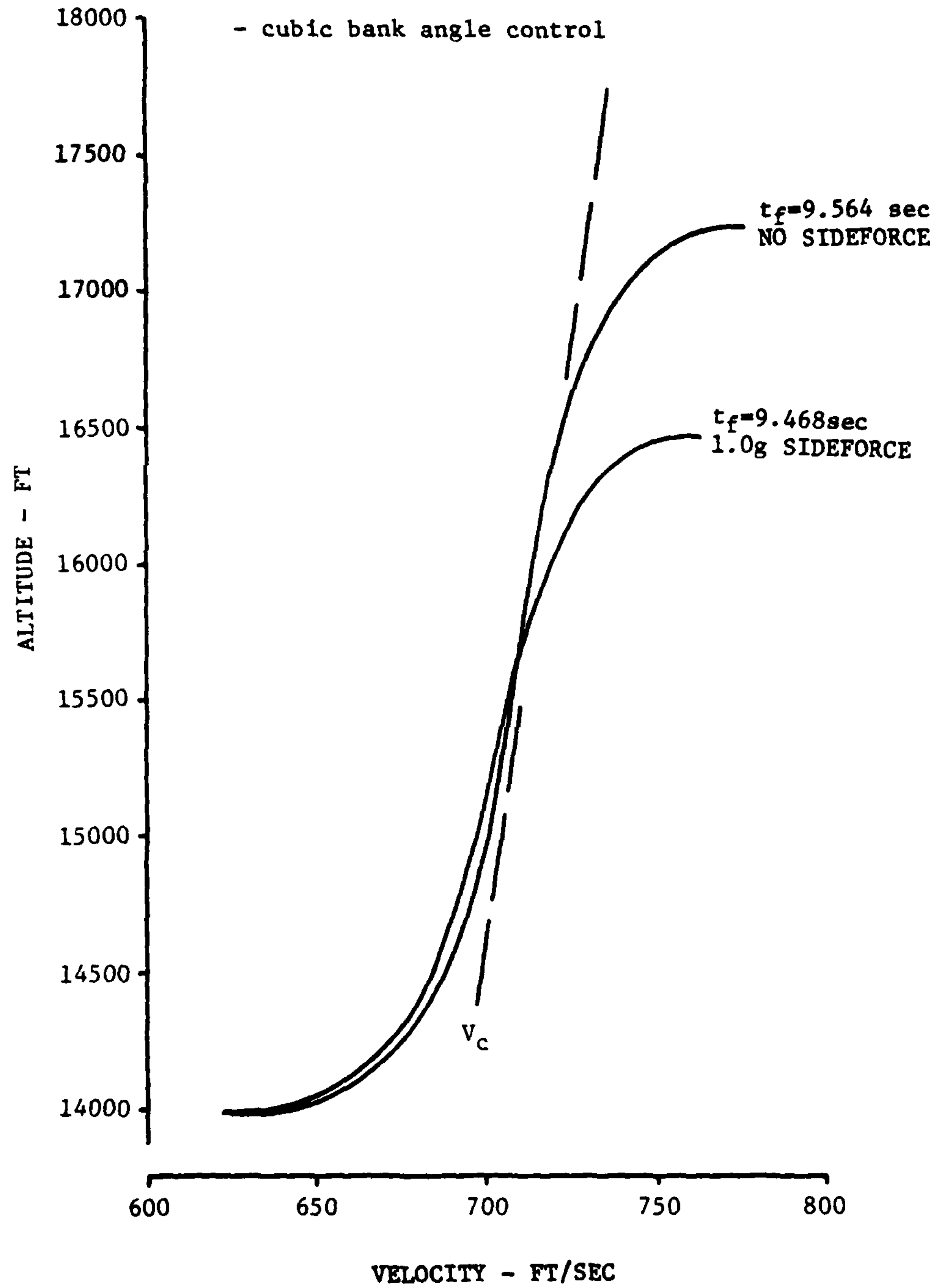

Fig. 7. Altitude vs. Velocity Trajectory for V_i = 621 ft/sec

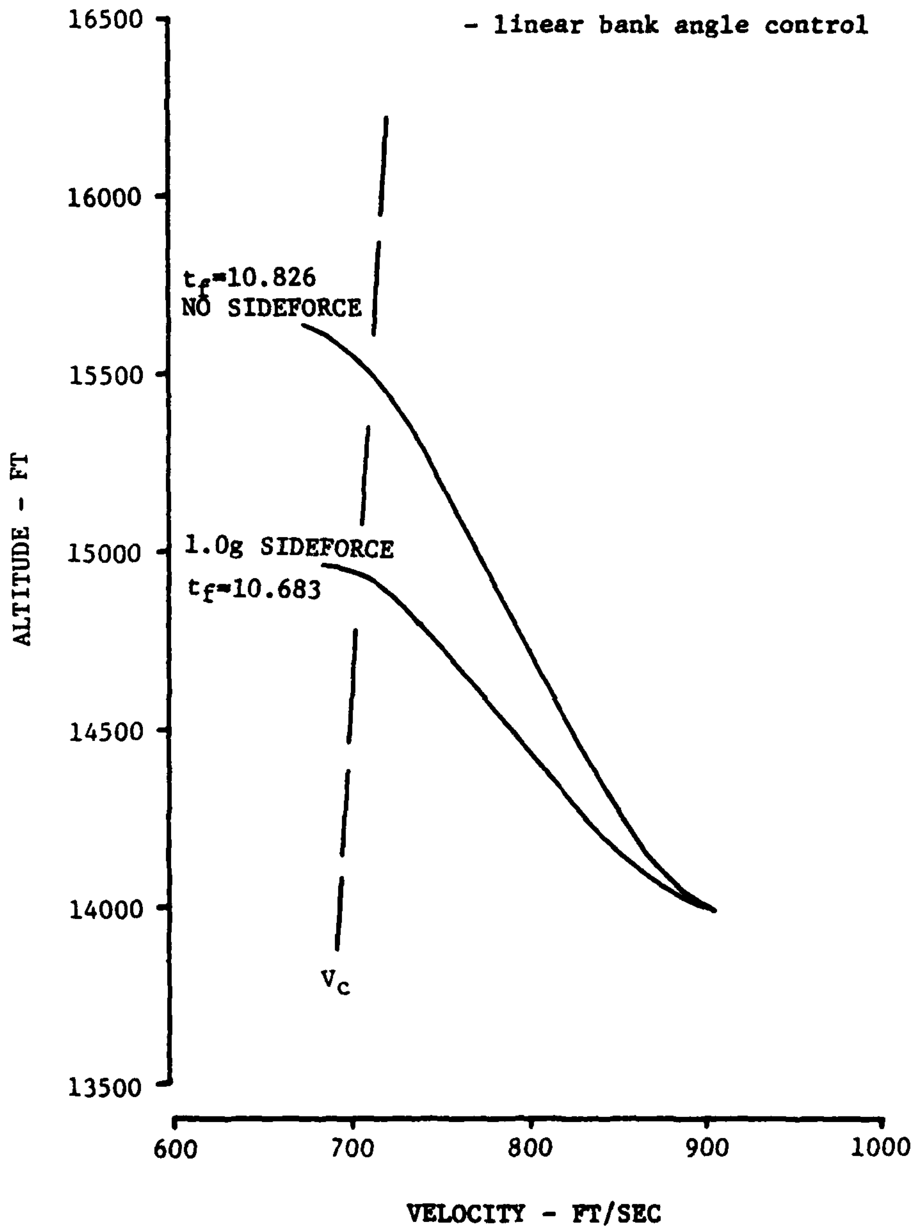

Fig. 8. Altitude vs. Velocity Trajectory for V_i = 903 ft/sec

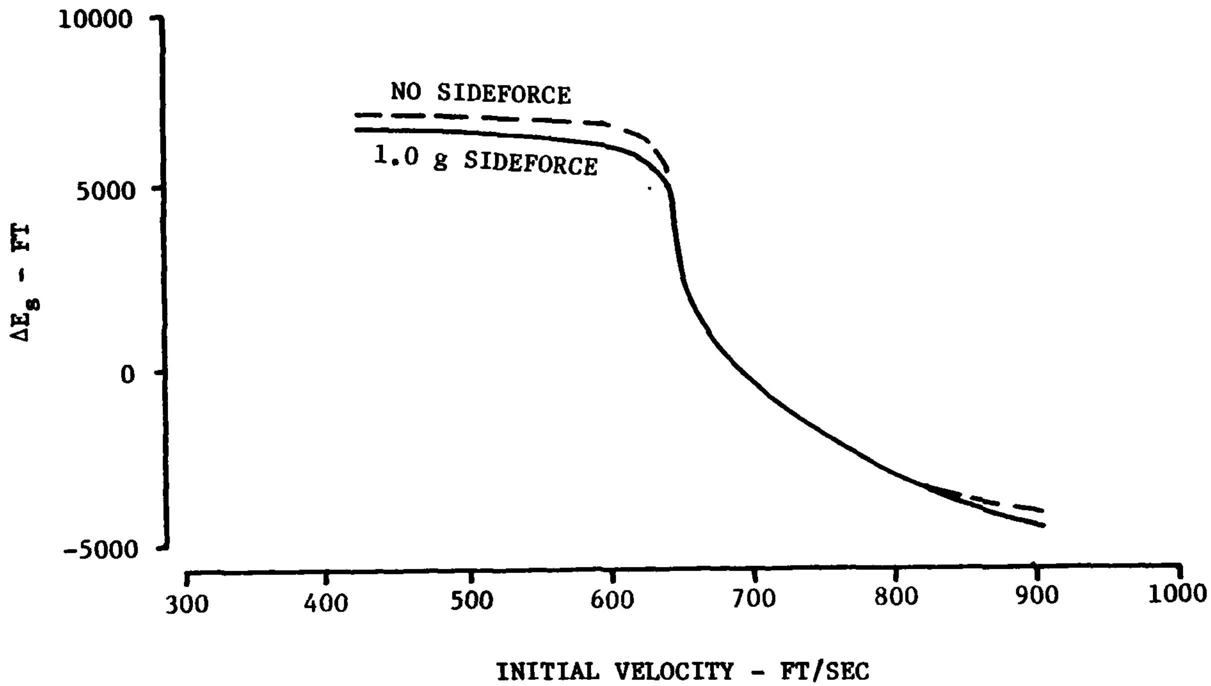

Fig. 9. Energy Gained in Turn vs. Initial Velocity

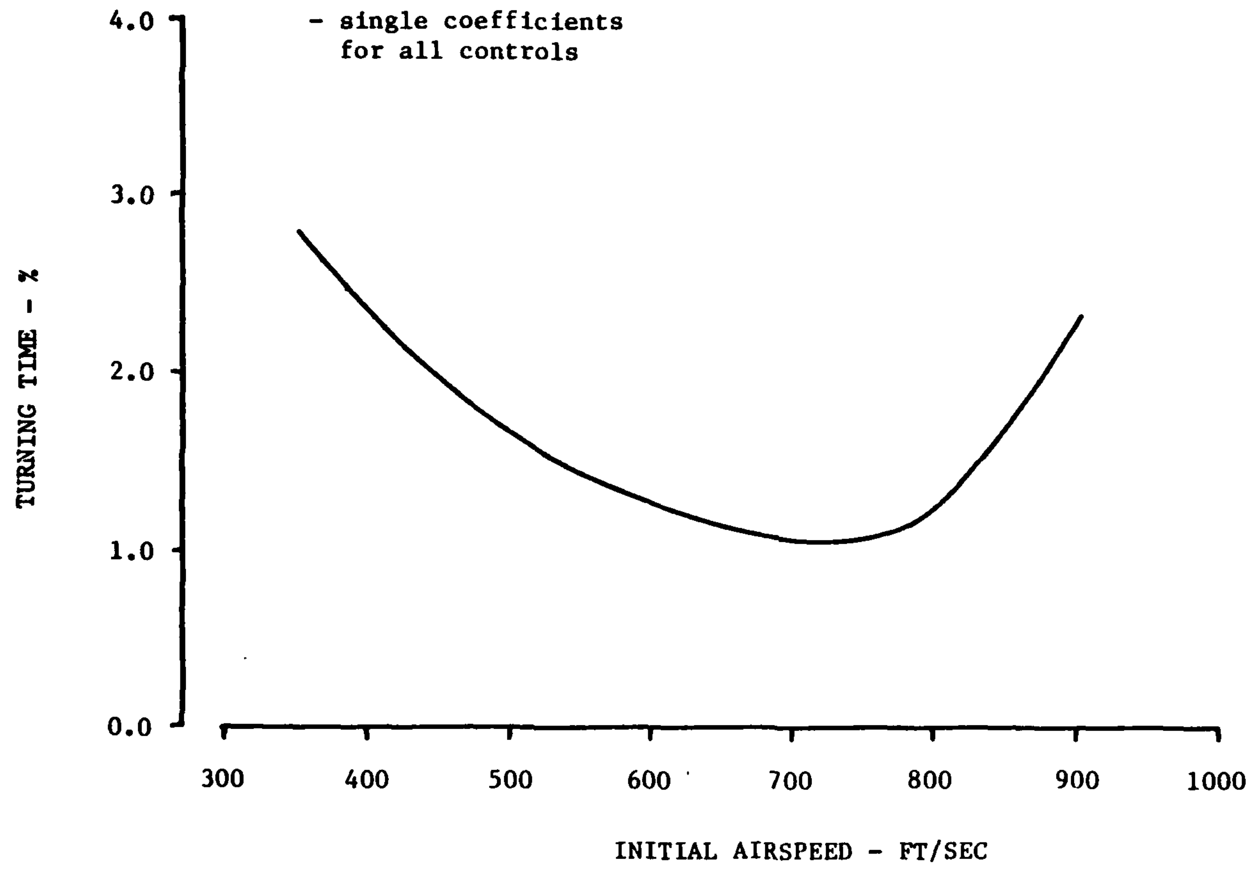

Fig. 10. Turning Time Reduction due to Sideforce

the turn. This is apparently caused by the added induced drag of the sideforce generator. And, since the thrust control is against the upper limit, no additional thrust is available to counteract the additional drag. Likewise, the high initial velocity case shows a slight loss of energy due to sideforce. In this case the thrust is at its lower limit in order for the aircraft to quickly slow to the corner velocity. In this respect the added drag due to sideforce aids the deceleration, but it also causes a slightly higher energy loss.

In summary of the primary function of sideforce, Figure 10 shows the percent of turning time saved by the use of direct sideforce. Additionally, the control coefficients, final time and change in specific energy are tabulated in the appendix.

Variations in Parameters

The general results given by Well and Berger (7) indicates there exists some initial velocity below the corner velocity for which the optimum maneuver is a split-S. That is, a roll of 180 degrees followed by a pull through in the vertical plane. The strategy in this technique is to use gravity, in conjunction with thrust, to accelerate the aircraft to the corner velocity as quickly as possible. And, as can be seen in Figure 3, the aircraft which can get to its corner velocity the quickest and maintain it will complete the turn the quickest. On the other side of the corner velocity, Well and Berger (7) found for some initial speed above the corner velocity, the optimum maneuver will be a vertical pull-up. The physical reasoning again being to use gravity to help slow the aircraft to the corner velocity.

The curves in Figures 11 and 13 show the bank angle time history, the vertical plane trajectory, and the altitude/velocity trajectory, respectively. Of particular interest is Figure 12 because it indicates the turn is performed in nearly a constant plane. As can be seen, even for the wide variation in initial velocities, the plane of the turn approaches vertical only for very low or very high initial airspeeds. Extrapolating from the values of Figure 12 indicates the lower velocity would be well below stall speed (256 ft/sec @ h = 13,990 ft) and the upper velocity must be far into the compressible flow region where the aircraft model is inadequate. The reason the maneuver plane angle varies so little with initial airspeed can be seen in Figure 14. This figure shows velocity and flight path angle as a function to time. In the beginning of the turn, the flight path angle is negative and the aircraft is accelerated toward the corner velocity with the aid of gravity. Upon, or slightly before reaching the corner velocity, the optimal solution indicates γ should bottom out and become more positive in order to prevent the aircraft from accelerating too far past the corner velocity. If the plane of the turn were steeper, the γ curve in Figure 14 would become more deeply cupped and would increase the acceleration due to gravity during the initial part of the turn. With the increases in acceleration, however, the corner velocity would be attained quicker, much before the γ curve bottomed out. Hence, the aircraft would be accelerated well beyond the corner velocity thereby increasing the turning time.

In an effort to try and match the general results of Well and Berger and represent an aircraft of more modest performance, the available thrust was reduced by half. This would force the optimiza-

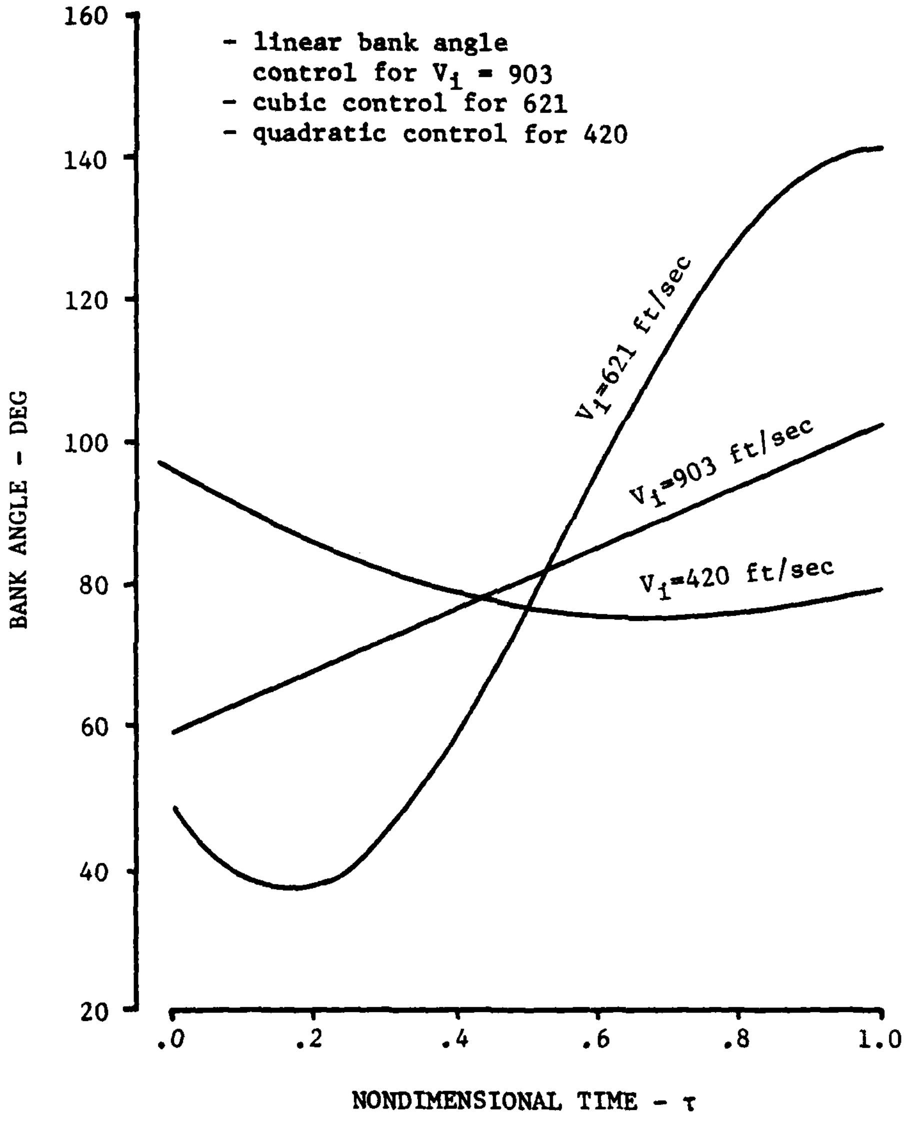

Fig. 11. Bank Angle Time History for Nominal Aircraft

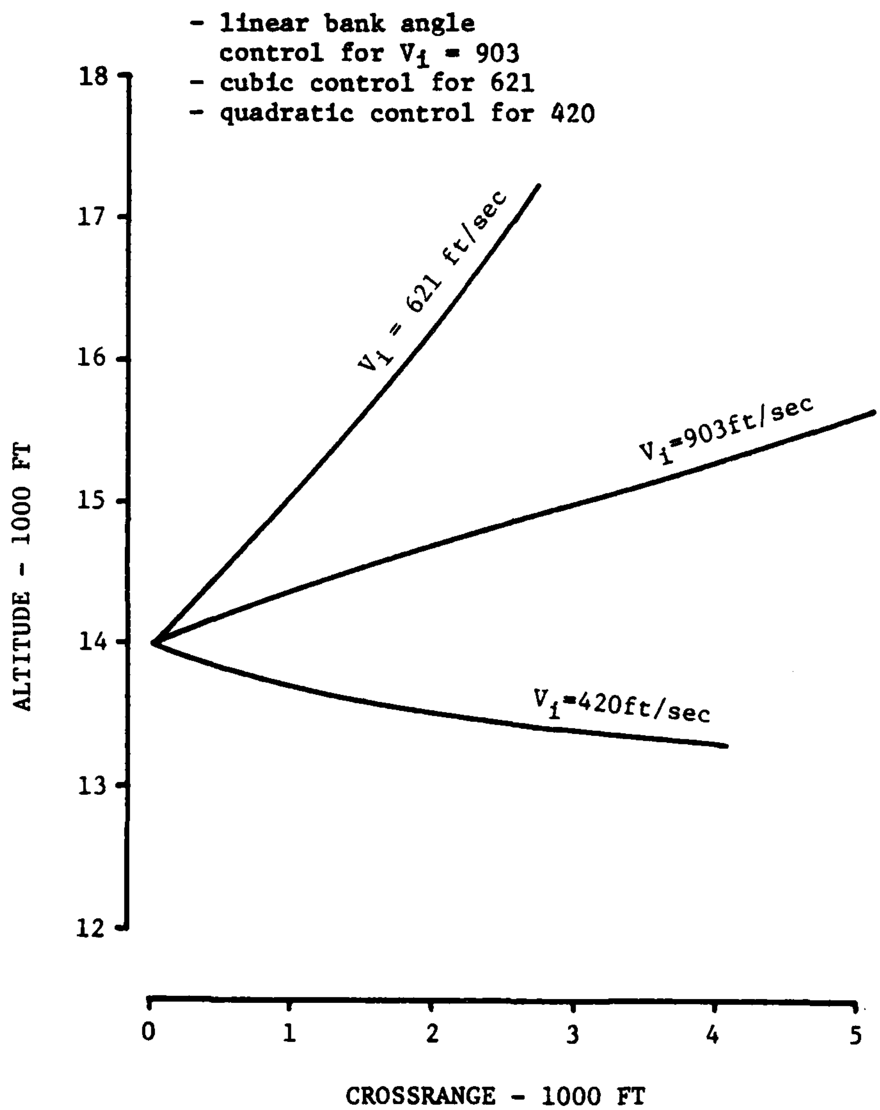

Fig. 12. Altitude vs. Crossrange Trajectory for Nominal Aircraft

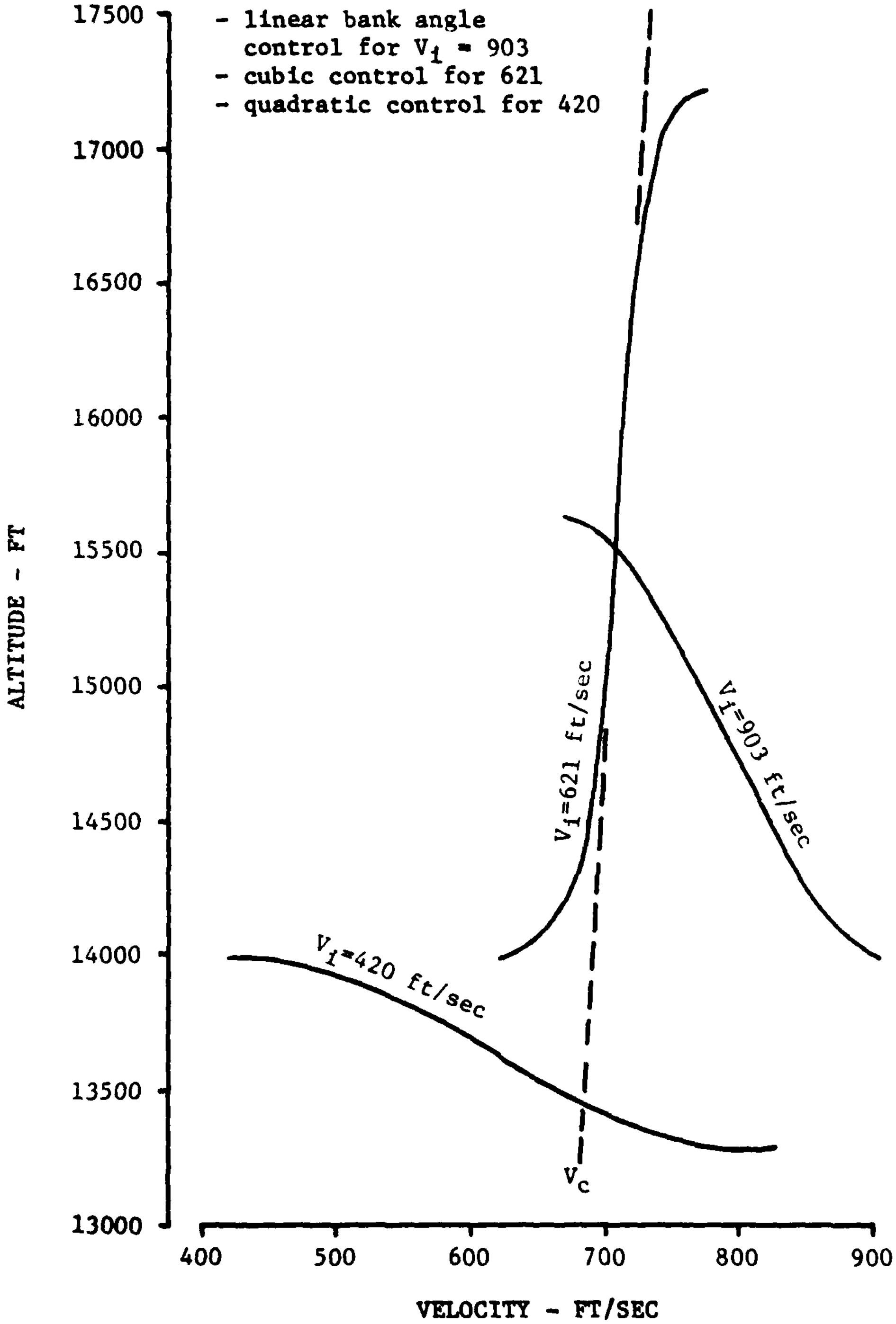

Fig. 13. Altitude vs. Velocity Trajectory for Nominal Aircraft

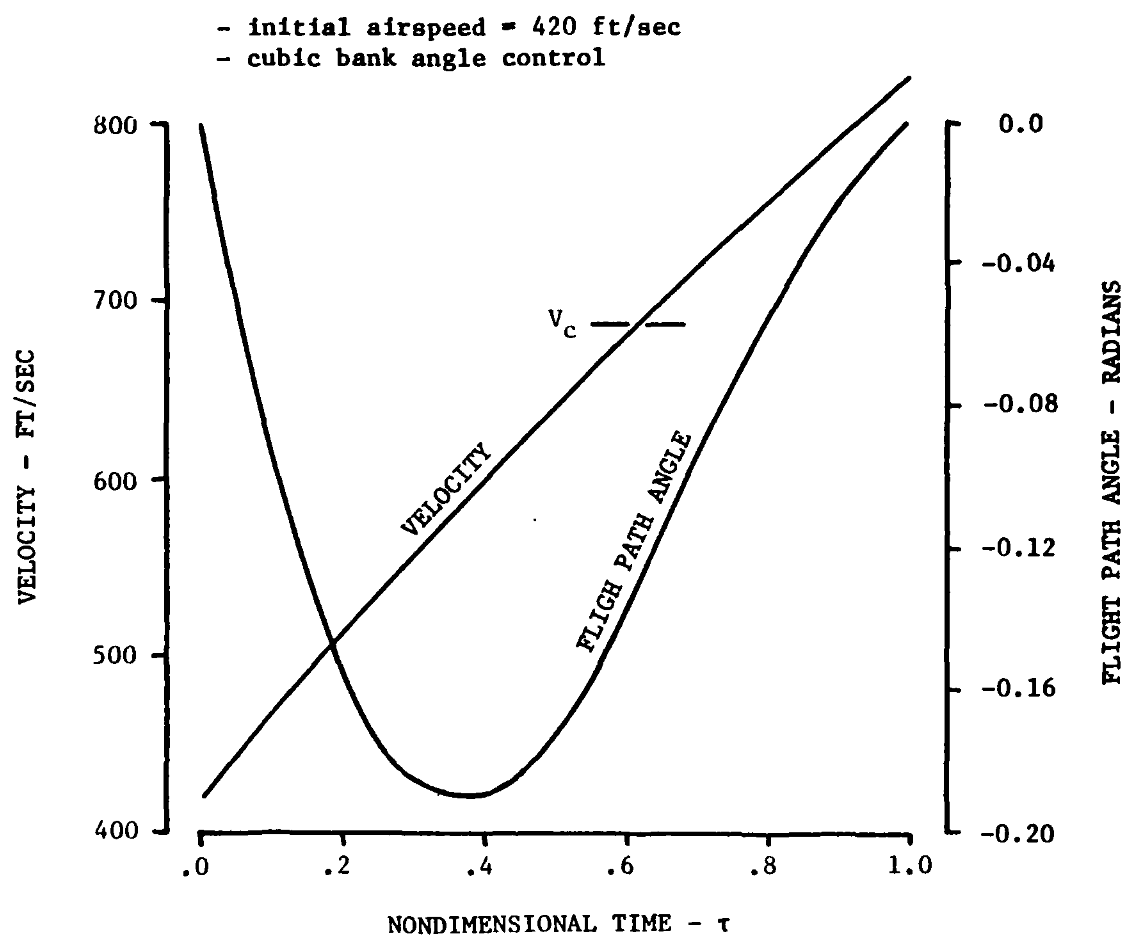

Fig. 14. Velocity and Flight Path Angle Time Histories

tion technique to use gravity more heavily and hence steepen the maneuver plane angle. The altitude/crossrange trajectory is shown in Figure 15. In comparison to the curves for the nominal aircraft in Figure 12, the turning planes are steeper. However, they still do not approach vertical for velocities within the scope of the aircraft model. Also note that changing the maximum thrust limit does not affect the high initial velocity case since the thrust is set to zero throughout the turn. One explanation for the weak effect thrust has on the turning plane can be seen Figure 16. This plot shows the drag to weight ratio of the aircraft for a given normal load factor as a function of velocity. The two horizontal lines at values of 1.5 and 0.75 represent the thrust available to counteract the drag. It is apparent that the nominal aircraft model requires very little thrust to maintain even its maximum load factor. Hence the aircraft tended to use its large excess thrust to attain corner velocity without much dependence on gravity.

A survey of similar aircraft was made to determine if the drag model was unrealistic. The drag polars in Figure 16 show the nominal model and a more realistic one derived from actual data. It is interesting to note the form drag or C_{D_o} is the same in both cases. However, the drag due to lift coefficient, K_1, was increased by more than four times. This change greatly affected the drag curves of Figure 16 as shown in Figure 18. Again the bank angle time histories, vertical plan trajectories, and altitude/velocity trajectories are illustrated in Figures 19 through 21. Only the thrust to weight ratio of 1.5 was used with the new drag model.

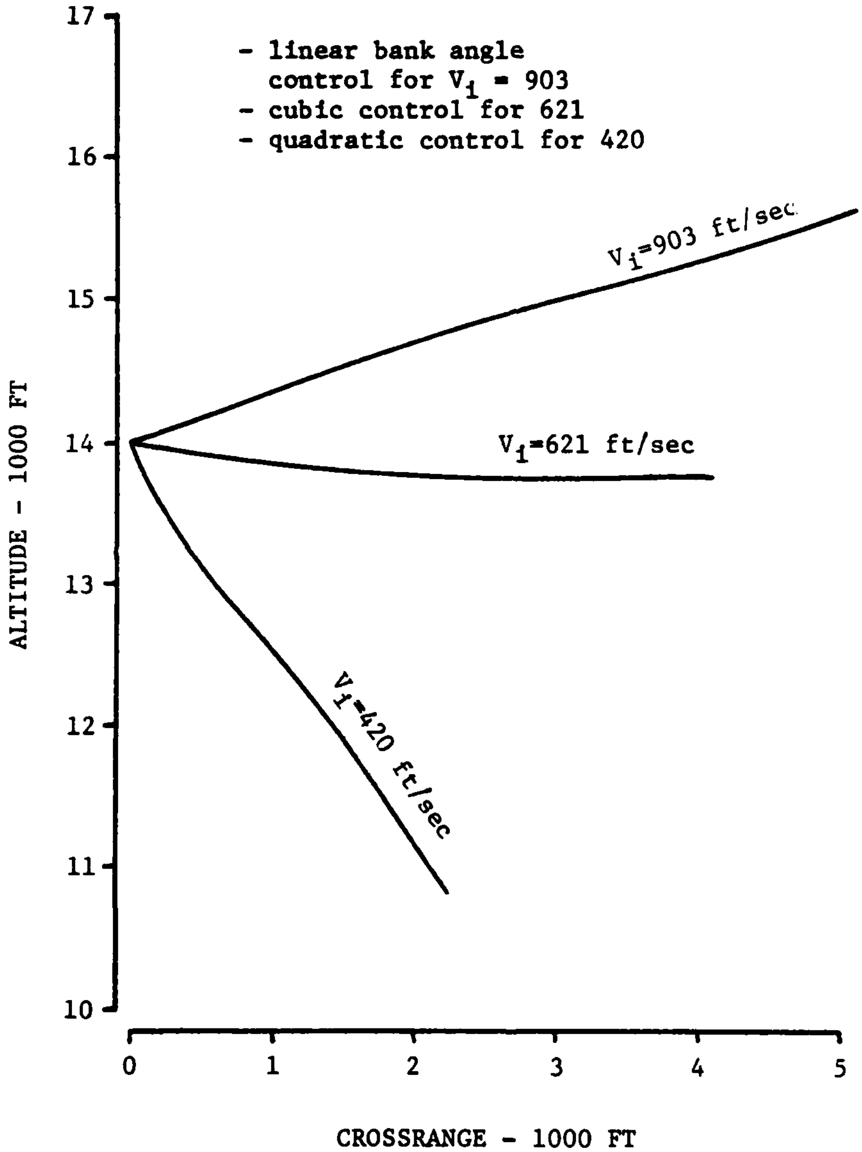

Fig. 15. Altitude vs. Crossrange Trajectories for T/W = .75

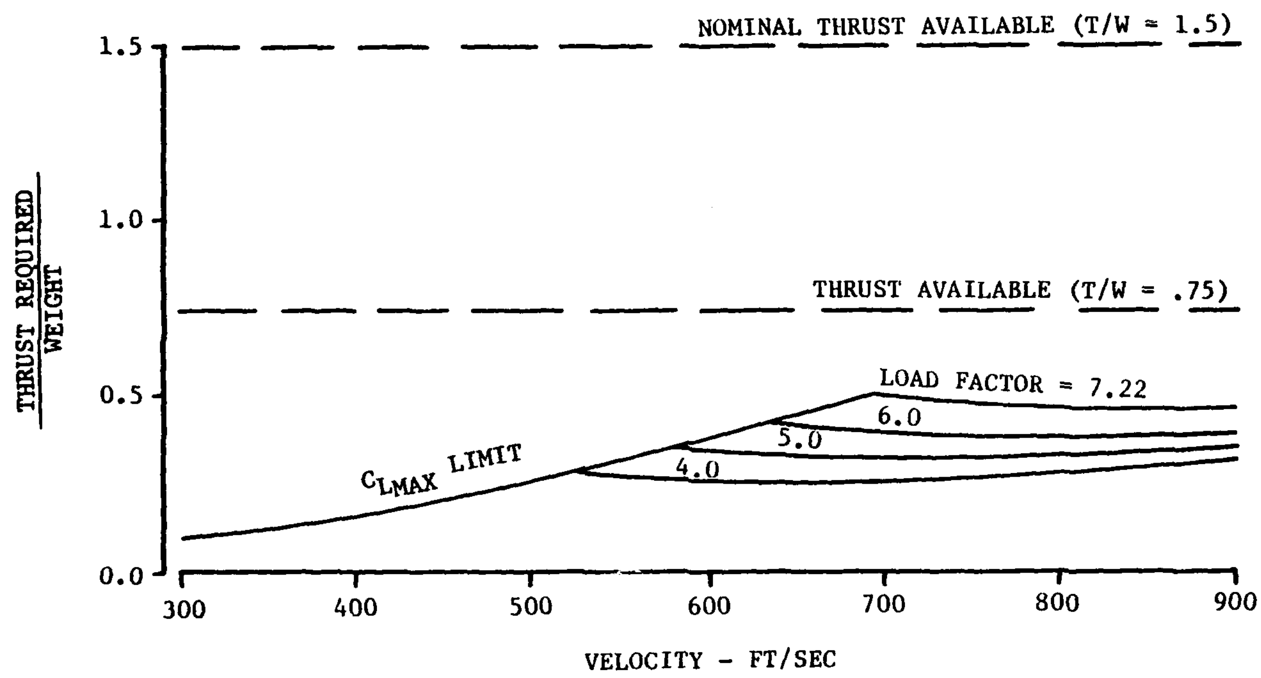

Fig. 16. Thrust Required/Available for Nominal Aircraft

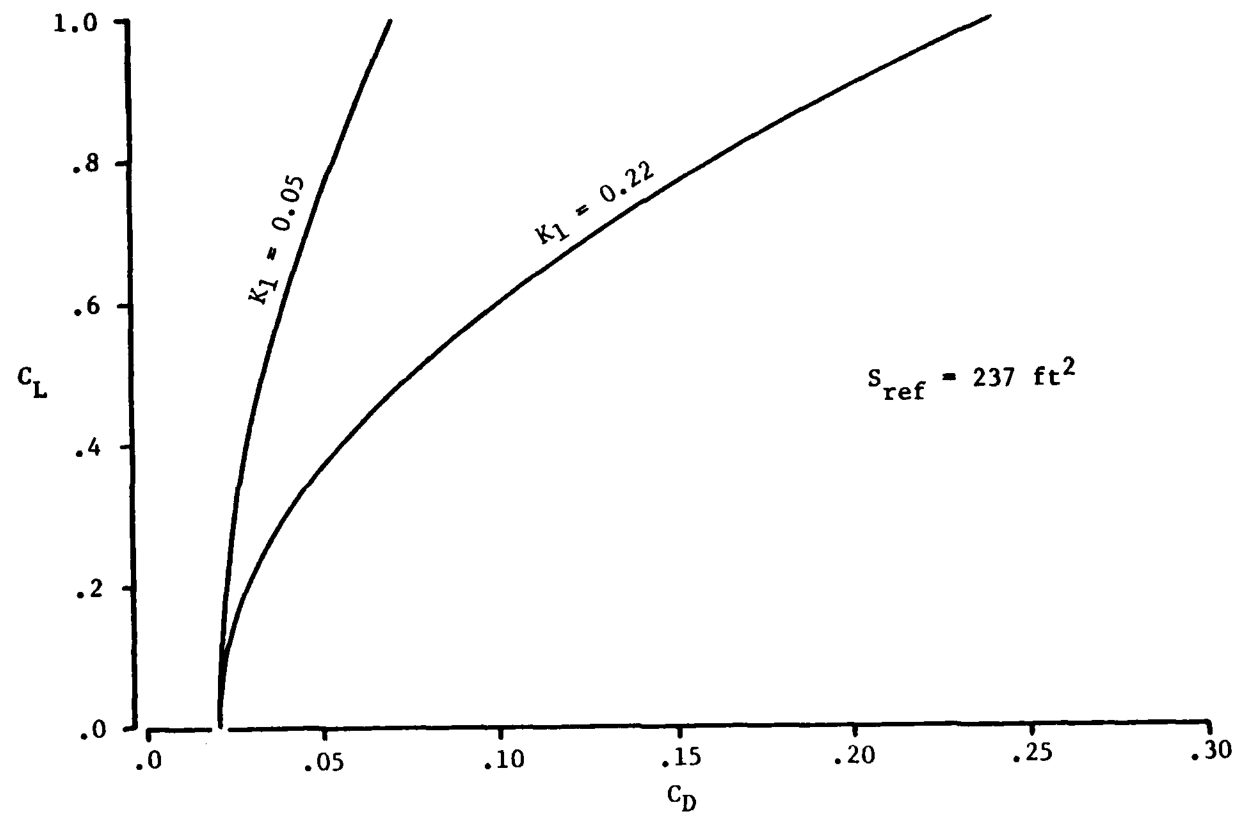

Fig. 17. Aircraft Drag Polars

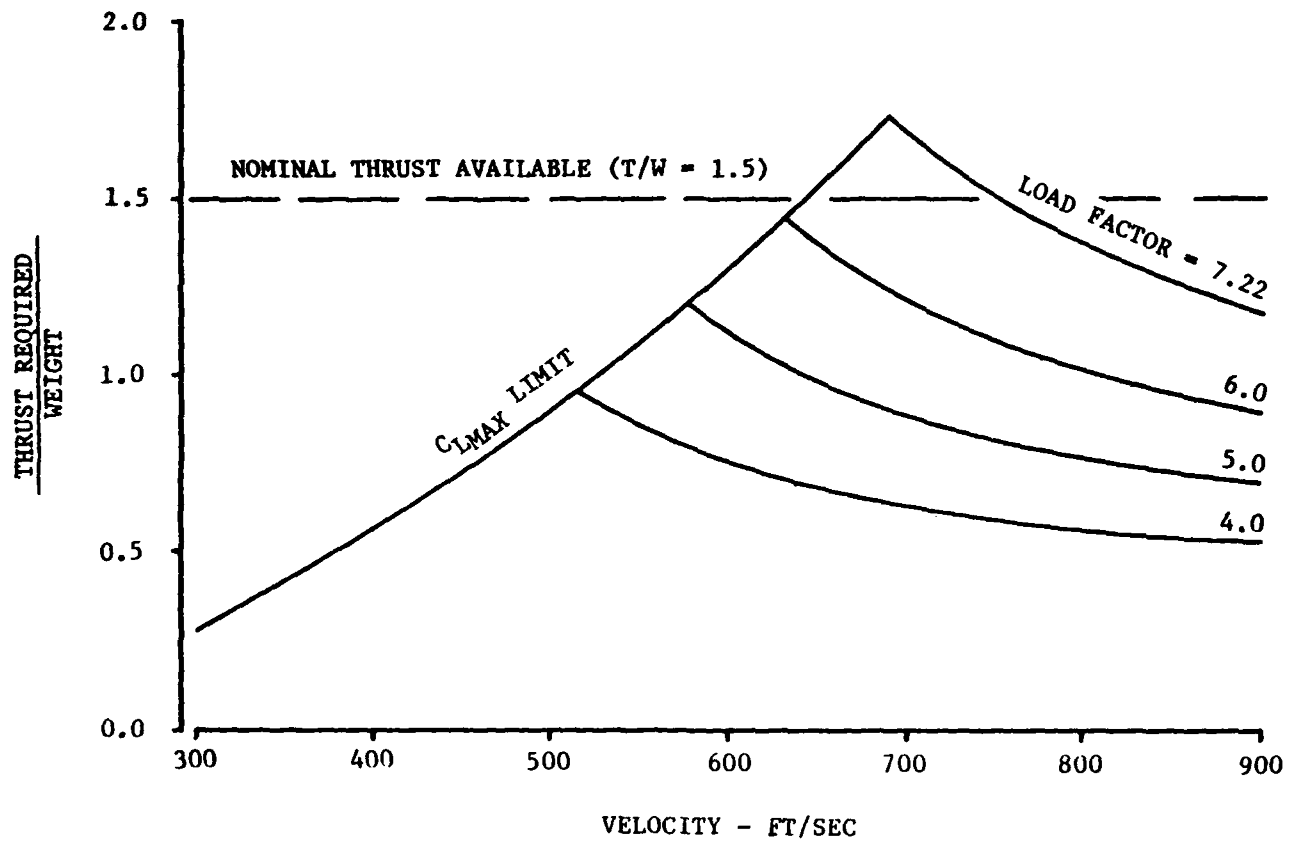

Fig. 18. Thrust Required/Available for K_1 = .22

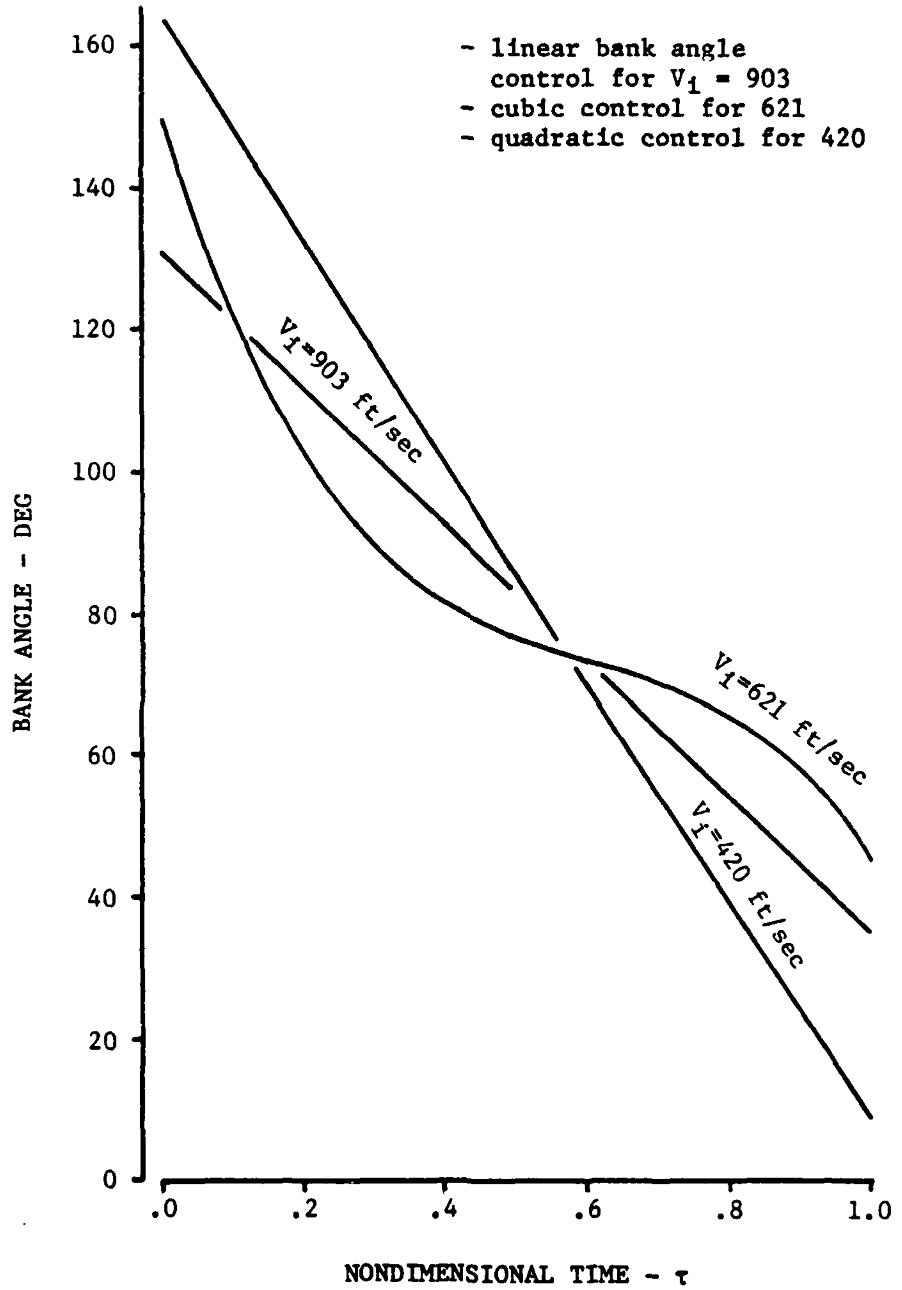

Fig. 19. Bank Angle Time History for K_1 = .22

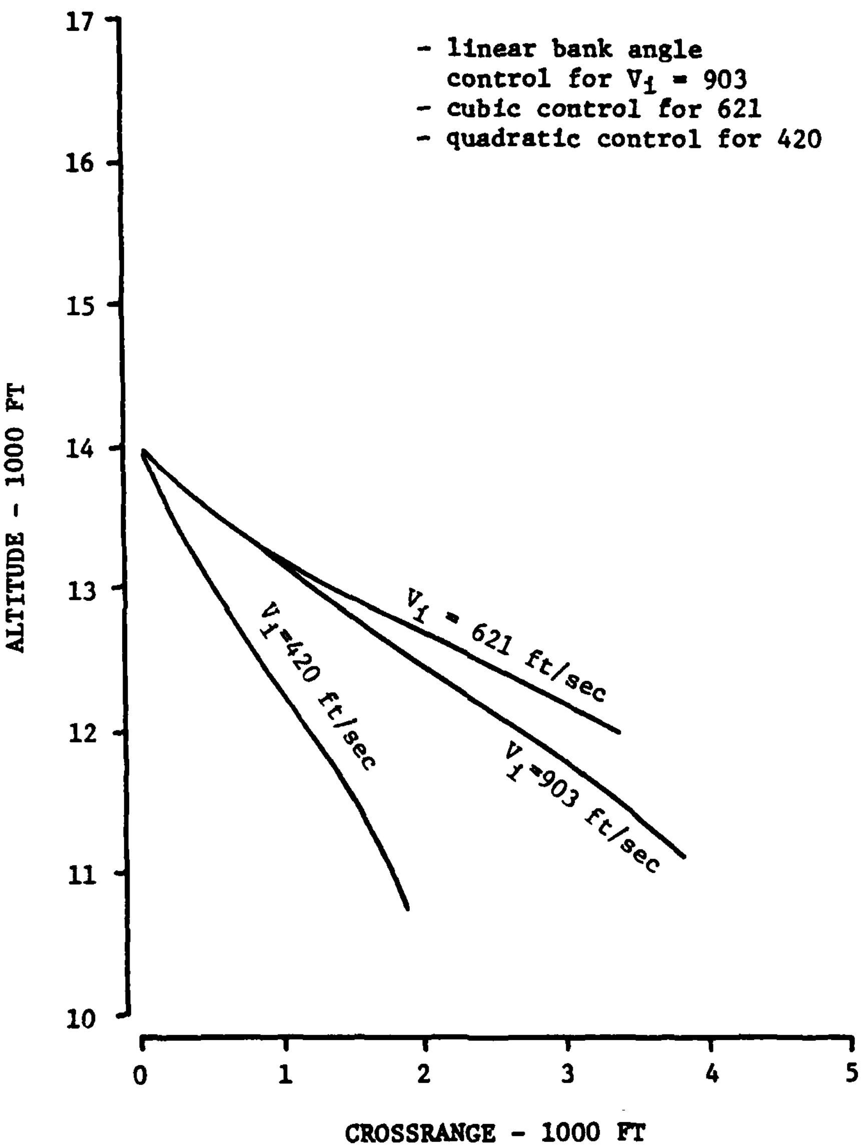

Fig. 20. Altitude vs. Crossrange Trajectory for K_1 = .22

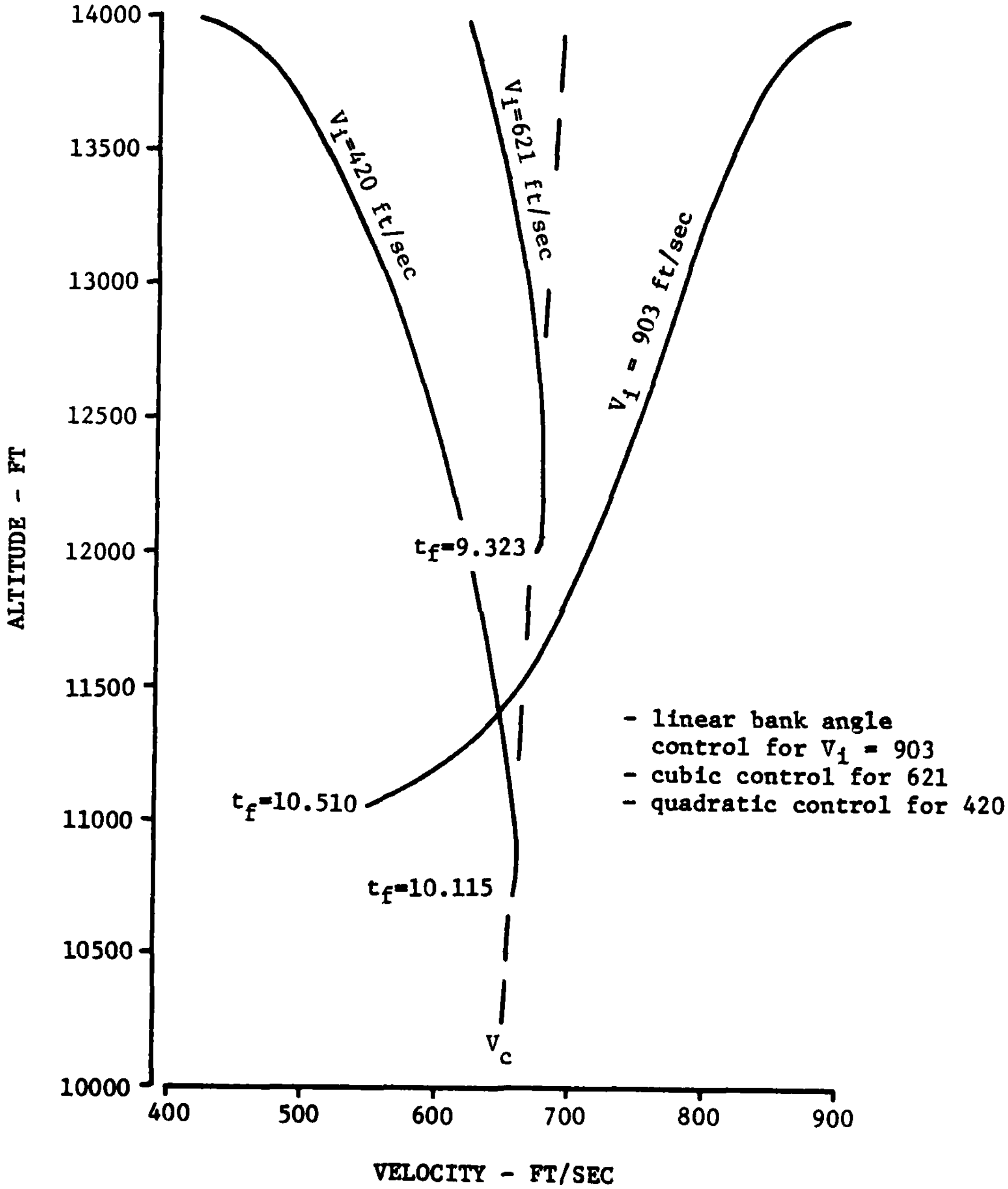

Fig. 21. Altitude vs. Velocity Trajectory for $K_1 = .22$

But Figure 20 shows that the turning plane is steeper for the low speed case and a vertical optimum turning plane is possible for a slightly slower initial airspeed. For the high speed case, the drag actually helps slow the aircraft and hence reduces the plane angle. Therefore, for the optimum maneuver to be a half-loop, the initial airspeed must be far beyond the upper bound of this study.

VII Conclusions and Recommendations

The conclusions on the benefits of using direct sideforce to reduce turning time based on the results of the previous chapter are:

1. The use of direct sideforce reduces the turning time from 1% to 3%.

2. The further the initial airspeed is from the corner velocity, the more the turning time is reduced.

3. In all cases considered, the optimal use of sideforce to minimize time is to use maximum sideforce for the full duration of the maneuver.

4. The additional energy gained or lost due to the use of sideforce depends on the initial conditions of the maneuvers as well as the characteristics of the aircraft.

From these conclusions it appears the use of direct sideforce is an effective way of reducing the turning time; however, the reduction is not large for the conditions studied here. But, under certain circumstances the reduction in turning time is accompanied by a loss in energy, therefore, judgement would have to be exercised by the pilot as to whether the energy loss could be tolerated.

The secondary effort of attempting to match the results of Well and Berger (2) was moderately successful. It appears the trends are the same in that turns initiated below the corner velocity tended to use gravity to accelerate to V_c and those initiated above the corner velocity tended to do the opposite. However, for the cases and aircraft parameters considered here, no threshold velocities for vertical turning planes could be found.

An aircraft's velocity vector can only be turned by an acceleration componant normal to the flight path. A normal force componant, therefore, is necessary. And, the larger this force, the less time it requires to perform a turn. There are two ways to increase this normal force. The first is through the use of speed control. This technique accelerates the aircraft as quickly as possible to the airspeed which maximizes the normal force of lift. An example is the use of reversible thrust by Johnson (4). Although the thrust does not directly aid in turning the aircraft, it accelerates or decelerates the aircraft to the velocity which maximizes lift, thereby minimizing turning time. The second way of increasing the normal force is by applying an additional force generator such as sideforce fins or vectored thrust. This technique can directly increase the resultant normal force over a wide range of velocities. This conclusion is supported by Figure 3 where the final turning time is reduced nearly uniformly over a wide initial velocity band.

It is recommended that the minimum time to turn problem be considered for an aircraft which incorporates both forms of control. The longitudinal acceleration could take the form of reversible thrust or aerodynamic fins which could be symmetrically deployed. The normal acceleration could be augmented by either a vectorable nozzle for direct lift from the engine or by aerodynamic sideforce surfaces.

Appendix A: Data Tables

V_{inial}		420 ft/sec		621 ft/sec		903 ft/sec	
	A	WITHOUT SIDEFORCE	WITH SIDEFORCE	WITHOUT SIDEFORCE	WITH SIDEFORCE	WITHOUT SIDEFORCE	WITH SIDEFORCE
BANK ANGLE COEFFICIENTS	B_1	1.4384	1.2570	1.5016	1.3336	1.4138	1.2830
	B_2	-0.1455	-1.2798	1.0103	0.7728	0.3816	0.2166
	B_3	0.0953	0.7114	0.1605	0.1012		
	B_4			-0.2046	-0.0813		
THRUST COEFFICIENT	C_1	1.0	1.0	1.0	0.9266	0.0	0.0
ANGLE OF ATTACK COEFFICIENT	D_1	0.2 FOR $V \le V_c$, $\alpha = (62287/\sigma V^2)$ FOR $V > V_c$					
SIDEFORCE COEFFICIENT	E_1		-1.0		-1.0		-1.0
SPECIFIC ENERGY GAINED DURING MANEUVER	E_s	7155 ft	6711 ft	6606 ft	5530 ft	-4009 ft	-4473 ft
LAGRANGE MULTIPLIERS	ν_1	-0.7648	-0.9305	2.1213	1.6692	0.3745	0.2634
	ν_2	-3.4781	-3.3762	-1.9620	-2.4437	-3.1798	-3.1052
FINAL TIME	t_f	10.5694	10.3565	9.5637	9.4684	10.8261	10.6825

$V_{initial}$		420 ft/sec		621 ft/sec		903 ft/sec	
$[T/W]_{max}$		0.75	1.5	0.75	1.5	0.75	1.5
K_1		0.05	0.22	0.05	0.22	0.05	0.22
BANK ANGLE COEFFICIENTS	B_1	1.5300	1.5050	1.4645	1.5132	1.4138	1.4526
	B_2	-1.2378	-1.3545	-0.0683	-0.7572	0.3816	-0.8408
	B_3	0.0917	0.0045	0.1124	0.1795		
	B_4			-0.0222	-0.1406		
THRUST COEFFICIENT	C_1	1.0	1.0	1.0	1.0	0.0	0.0
ANGLE OF ATTACK COEFFICIENT	D_1	0.2 FOR $V \leq V_c$; $\alpha = (62287/\sigma V^2)$ FOR $V > V_c$					
LAGRANGE MULTIPLIERS	ν_1	-2.9311	-2.9296	-0.3310	-0.9924	0.3745	-3.0546
	ν_2	-1.9365	-1.5034	-3.0410	-3.0237	-3.1789	-2.6690
FINAL TIME	t_f	10.5748	10.1153	9.6101	9.3231	10.8261	10.5100

Appendix B: Program Listing

```
      PROGRAM SUBOPT(INPUT,OUTPUT)
      DIMENSION A(22),AA(22),DA(22),GA(22),M(2),MA(2,22),MAAI(22,22),
     1MAA2(22,22),X(6),DX(6),XX(6),XP(6),XP1(6),XP2(6),XP3(6),XP4(6),
     2 DEL(22),MAMAT(2,2),MAMATI(2,2),MAGAT(2),FAT(22),FAA(22,22),
     3 FAAI(22,22),MAFAAI(2,22),B(2,2),BI(2,2),C(2),D(22),PNU(2),
     4 DPNU(2),PNUMA(22),TOT(5),Z(7),E(22),FA(22),FFAA(22,22),
     5 FFAAI(22,22),TT(7)
      COMMON/MISC/NPH,NPI,NA,NS,TF,DT,N,S,LL,NN,PP,QQ,AA,TOT
      COMMON/MISC1/BA,PI,SIG,ALF
      REAL M,M1,M2,M3,M4,MM1,MM2,MM3,MM4,MM5,MM6,MM7,MM8,MA,MAAI
      REAL MAA2,MAMAT,MAMATI,MAGAT,MAFAAI,NORMA,NORMM,NORMP
C
C  PROGRAM SUBOPT IS A SUBOPTIMAL CONTROL TECHNIQUE USED TO FIND AN
C  APPROXIMATE SOLUTION TO AN OPTIMAL CONTROL PROBLEM.  THE SOLUTION TO
C  MOST OPTIMAL CONTROL PROBLEMS IF THE CONTROLS CANNOT BE SOLVED
C  ANALYTICALLY IS TO GUESS THE CONTROLS AND THE LAGRANGE MULTIPLIERS
C  AND SEE IF END CONDITIONS ARE MET AND THE PERFORMANCE INDEX MINIMIZED
C  THE SUBOPTIMAL CONTROL TECHNIQUE ASSUMES THE CONTROLS ARE A LINEAR
C  COMBINATION OF POLYNOMIALS WITH UNKNOWN COEFFICIENTS.  IN SO DOING
C  THE PROBLEM IS CHANGED FROM A FUNCTIONAL MINIMIZATION PROBLEM TO A
C  PARAMETER OPTIMIZATION PROBLEM.
C
C
C  A    = MATRIX OF PARAMETERS (FINAL T,PHI COEF,PI COEF,ALPHA COEF)
C  DA   = MATRIX OF PARAMETER CHANGES (NP X 1)
C  NE   = NUMBER OF DIFFERENTIAL CONSTRAINTS (STATE EQUATIONS)
C  NPH  = NUMBER OF PHI COEFFICIENTS
C  NA   = NUMBER OF ALPHA COEFFICIENTS
C  NPI  = NUMBER OF PI COEFFICIENTS
C  NS   = NUMBER OF SIDE FORCE COEFFICIENTS
C  NP   = TOTAL NUMBER OF PARAMETERS (1+NPH+NPI+NA+NS)
C  NC   = NUMBER OF END CONDITIONS TO BE SATISFIED
C  P    = SCALING FACTOR FOR PERFORMANCE INDEX
C  Q    = SCALING FACTOR FOR END CONDITION CONSTRAINTS (M)
C  G    = SCALAR PERFORMANCE INDEX
C  GA   = PARTIAL DERIVATIVES OF G WRT A'S (1 X NP)
```

```
C  M    = MATRIX OF PRESCRIBED FINAL CONDITIONS (NC X 1)
C  MA   = PARTIAL DERIVATIVES OF M WRT A'S (NC X NP)
C  MAA  = SECOND PARTIAL DERIVATIVES OF M WRT A'S (2 - NP X NP)
C  X    = STATE VARIABLES (X,Y,H,VEL,THETA,PSI)
C  DX   = DERIVATIVE OF STATE VARIABLES WRT (T/TF)
C  F    = AUGMENTED PERFORMANCE INDEX (G + PNUT*M)
C  FA   = PARTIAL DERIVATIVE OF F WRT A'S (1 X NP)
C  FAT  = PARTIAL DERIVATIVES OF F WRT A'S TRANSPOSED (NP X 1)
C  FAA  = SECOND PARTIAL DERIVATIVE OF F WRT A'S (NP X NP)
C  PNU  = LAGRANGE MULTIPLIERS (NC X 1)
C  DPNU= CHANGE IN LAGRANGE MULTIPLIERS (NC X 1)
C
C
C  PARAMETERS, INITIAL CONDITIONS AND GUESSES
C
      NE=6
      N=NE
      X(1)=X(2)=X(5)=X(6)=0.0  $  X(3)=13990.0  $  X(4)=420.0
      NPH=3
      NPI=1
      NA=1
      NS=0
      NP=NPH+NPI+NA+NS+1
      NC=2
      FGNORM=10.0
      FNORMA=10.0
      ITER=0
      MAX=15
      DELTA=1.0E-4
      CC1=1.0E-4
      CC2=1.0E-2
      CC3=1.0E-8
      CC4=1.0E-4
      CC5=1.0E-4
      P=1.0
      P=.1
```

```
      Q=1.0
      V=1.0
C     INITIAL GUESSES
C     FINAL TIME COEFFICIENTS
      A(1)=10.12004417
C     BA COEFFICIENTS
      A(2)=1.506341517
      A(3)=-1.437617675
      A(4)=-.003885144245
C     THRUST COEFFICIENTS
      A(5)=1.0
C     ALPHA COEFFICIENTS
      A(6)=0.2
      PNU(1)=-2.825251987-.1992591
      PNU(2)=-1.383341708-.06010897
      DPNU(1)=DPNU(2)=0.0
50    FORMAT(5X,"PNU'S ",4E17.10)
51    FORMAT(1X,"A MATRIX  ",7E17.10)
52    FORMAT(1X,5E15.7)
53    FORMAT(100X,"NORM OF M'S" ,E15.7)
54    FORMAT(1X,"FA MATRIX ",8E15.7)
C....NOTE NC=2 HERE...........................................
55    FORMAT(5X,"DPNU'S ",2E15.7,10X,"NORM OF DPNU'S ",E15.7)
56    FORMAT(1X,"DA MATRIX ",8E15.7)
57    FORMAT(100X,"NORM OF DA'S ",E15.7)
C
C  DETERMINING M MATRIX BY INTEGRATING DIFFERENTIAL CONSTRAINTS
C
      DO 1 I=1,NP
1     DA(I)=0.0
1000  DO 2 I=1,NP
      A(I)=A(I)+DA(I)
2     AA(I)=A(I)
      ITRAT=0
      PRINT*," "
      PRINT*,"ITERATION NUMBER   ",ITER,"  P= ",P,"      DELTA= ",DELTA
```

```
      PRINT*," "
      DO 3 I=1,NE
    3 XX(I)=X(I)
      S=1.0
      N3=1
      N4=1
      N5=1
      LL=0
      DT=.01
      T=0.0
      TF=1.0
      CALL RK45(T,XX)
      M(1)=XX(5)-1.0E-10
      M(2)=XX(6)-3.141592654
      ITER=ITER+1
      PRINT*," "
      PRINT 51,(A(I),I=1,NP)
      PRINT*," "
      DO 4 I=1,NC
      PNU(I)=PNU(I)+DPNU(I)
    4 PRINT*,"M(",I,")= ",M(I)
      NORMM=0.0
      DO 5 I=1,NC
    5 NORMM=NORMM+M(I)**2
      NORMM=SQRT(NORMM)
      PRINT*," "
      PRINT 53,NORMM
C
C  DETERMINING MA AND MAA BY CENTRAL DIFFERENCES
C
      S=0.0
      DO 100 I=1,NP
      DO 6 J=1,NP
    6 AA(J)=A(J)
      DEL(I)=DELTA*A(I)
      IF(ABS(DEL(I)).LE.DELTA) DEL(I)=DELTA
      AA(I)=A(I)+DEL(I)
```

```
      DO 7 J=1,NE
    7 XP(J)=X(J)
      LL=0
      DT=.01
      T=0.0
      TF=1.0
      CALL RK45(T,XP)
      M1=XP(5)-1.0E-10
      M2=XP(6)-3.141592654
      AA(I)=A(I)-DEL(I)
      DO 8 J=1,NE
    8 XP(J)=X(J)
      LL=0
      DT=.01
      T=0.0
      TF=1.0
      CALL RK45(T,XP)
      M3=XP(5)-1.0E-10
      M4=XP(6)-3.141592654
      MA(1,I)=(M1-M3)/(2.0*DEL(I))
      MA(2,I)=(M2-M4)/(2.0*DEL(I))
      DO 100 K=1,I
      IF(K.EQ.I) GO TO 707
      DO 9 J=1,NP
    9 AA(J)=A(J)
      AA(I)=A(I)+DEL(I)
      DEL(K)=DELTA*A(K)
      IF(ABS(DEL(K)).LE.DELTA) DEL(K)=DELTA
      AA(K)=A(K)+DEL(K)
      DO 10 J=1,NE
   10 XP1(J)=X(J)
      LL=0
      DT=.01
      T=0.0
      TF=1.0
```

```
      CALL RK45(T,XP1)
      MM1= XP1(5)-1.0E-10
      MM2=XP1(6)-3.141592654
      DO 11 J=1,NP
   11 AA(J)=A(J)
      AA(I)=A(I)+DEL(I)
      AA(K)=A(K)-DEL(K)
      DO 12 J=1,NE
   12 XP2(J)=X(J)
      LL=0
      DT=.01
      T=0.0
      TF=1.0
      CALL RK45(T,XP2)
      MM3=(XP2(5)-1.0E-10)
      MM4=XP2(6)-3.141592654
      DO 13 J=1,NP
   13 AA(J)=A(J)
      AA(I)=A(I)-DEL(I)
      AA(K)=A(K)+DEL(K)
      DO 14 J=1,NE
   14 XP3(J)=X(J)
      LL=0
      DT=.01
      T=0.0
      TF=1.0
      CALL RK45(T,XP3)
      MM5=XP3(5)-1.0E-10
      MM6=XP3(6)-3.141592654
      DO 15 J=1,NP
   15 AA(J)=A(J)
      AA(I)=A(I)-DEL(I)
      AA(K)=A(K)-DEL(K)
      DO 16 J=1,NE
   16 XP4(J)=X(J)
      LL=0
```

```
      DT=.01
      T=0.0
      TF=1.0
      CALL RK45(T,XP4)
      MM7=XP4(5)-1.0E-10
      MM8=XP4(6)-3.141592654
      MAA1(I,K)=MAA1(K,I)=(MM1-MM3-MM5+MM7)/(4.0*DEL(I)*DEL(K))
      MAA2(I,K)=MAA2(K,I)=(MM2-MM4-MM6+MM8)/(4.0*DEL(I)*DEL(K))
      GO TO 100
707   MAA1(I,I)=(M1-2.0*M(1)+M3)/(DEL(I)**2)
      MAA2(I,I)=(M2-2.0*M(2)+M4)/(DEL(I)**2)
100   CONTINUE
C
C  DETERMINING GA
C
      GA(1)=1.0
      DO 17 I=2,NP
17    GA(I)=0.0
C
C  CALCULATING DPNU AND DA (SECOND ORDER TECH)
C
747   DO 24 I=1,NP
      DO 24 J=1,NP
24    FAA(I,J)=PNU(1)*MAA1(I,J)+PNU(2)*MAA2(I,J)
      ITRAT=ITRAT+1
      IF(ITRAT.GE.MAX) PRINT*,"ITERATIONS ",ITRAT
      IF(ITRAT.GE.MAX) GO TO 757
      DO 22 I=1,NP
      PNUMA(I)=0.0
      DO 23 J=1,NC
23    PNUMA(I)=PNUMA(I)+PNU(J)*MA(J,I)
22    FAT(I)=GA(I)+PNUMA(I)
      PRINT*," "
      PRINT 54,(FAT(I),I=1,NP)
C*******TAKE OUT EFFECTS OF CONTROLS OUTSIDE OF LIMITS*******
```

```
C**************AND SHRINK MATRICES*****************************
      IF(NN.EQ.1.AND.FAT(1+NPH+NPI).LE.0.0) N3=0
      IF(NN.EQ.-1.AND.FAT(1+NPH+NPI).GT.0.0) N3=0
      IF(PP.EQ.1.0.AND.FAT(1+NPH+NPI+NA).LE.0.0) N4=0
      IF(QQ.EQ.1.0.AND.FAT(NP).LE.0.0) N5=0
      IF(QQ.EQ.-1.0.AND.FAT(NP).GE.0.0) N5=0
      NI=NP
      IF(N3.NE.0) GO TO 62
      NP=NP-NPI
      IT=2+NPH
      DO 61 I=IT,NP
      FAT(I)=FAT(I+NPI)
      DO 60 J=1,NC
   60 MA(J,I)=MA(J,(I+NPI))
      DO 61 J=IT,NP
      DO 61 K=1,NI
      FAA(I,K)=FAA((I+NPI),K)
      FAA(K,I)=FAA(K,(I+NPI))
   61 FAA(I,J)=FAA((I+NPI),(J+NPI))
   62 IF(N4.NE.0) GO TO 65
      NP=NP-NA
      IT=2+NPH+NPI
      DO 64 I=IT,NP
      FAT(I)=FAT(I+NA)
      DO 63 J=1,NC
   63 MA(J,I)=MA(J,(I+NA))
      DO 64 J=IT,NP
      DO 64 K=1,NI
      FAA(I,K)=FAA((I+NA),K)
      FAA(K,I)=FAA(K,(I+NA))
   64 FAA(I,J)=FAA((I+NA),(J+NA))
   65 IF(N5.NE.0) GO TO 66
      NP=NP-NS
   66 CONTINUE
```

```
      DO 58 I=1,NP
      DO 58 J=1,NP
   58 FFAA(I,J)=FAA(I,J)
      CALL GAUSD(NP,1.0E-30,FFAA,FFAAI,JER,22)
      DO 48 I=1,NP
      DO 48 J=1,NP
   48 FAAI(I,J)=0.0
      DO 49 I=1,NP
      DO 49 J=1,NP
   49 FAAI(I,J)=FFAAI(I,J)
      DO 25 I=1,NC
      DO 25 J=1,NP
      MAFAAI(I,J)=0.0
      DO 25 K=1,NP
   25 MAFAAI(I,J)=MAFAAI(I,J)+MA(I,K)*FAAI(K,J)
      DO 26 I=1,NC
      DO 26 J=1,NC
      B(I,J)=0.0
      DO 26 K=1,NP
   26 B(I,J)=B(I,J)+MAFAAI(I,K)*MA(J,K)
      CALL GAUSD(NC,1.0E-30,B,BI,KER,NC)
      DO 27 I=1,NC
      C(I)=0.0
      DO 27 J=1,NP
   27 C(I)=C(I)+MAFAAI(I,J)*FAT(J)
      DO 28 I=1,NC
      DPNU(I)=0.0
      DO 28 J=1,NC
   28 DPNU(I)=DPNU(I)+BI(I,J)*(-P*C(J)+U*M(J))
      DO 29 I=1,NP
      D(I)=0.0
      DO 29 J=1,NC
   29 D(I)=D(I)+MA(J,I)*DPNU(J)
      DO 30 I=1,NP
      DA(I)=0.0
```

```
      DO 30 J=1,NP
30    DA(I)=DA(I)+FAAI(I,J)*(-P*FAT(J)-D(J))
C**********RE-EXPAND THE MATRICES******************************
C**********INSERTING ZERUS APPROPRIATELY*********************
      IF(N3.NE.0) GO TO 69
      NP=NP+NPI
      IT=NA+NS
      DO 67 I=1,IT
      DA(NP+1-I)=DA(NP+1-I-NPI)
      DO 67 J=1,NC
67    MA(J,(NP+1-I))=MA(J,(NP+1-I-NPI))
      DO 68 I=1,NPI
      DA(1+I+NPH)=0.0
      DO 68 J=1,NC
68    MA(J,(1+I+NPH))=0.0
69    IF(N4.NE.0) GO TO 72
      NP=NP+NA
      IF(NS.EQ.0) GO TO 75
      DO 70 I=1,NS
      DA(NP+1-I)=DA(NP+1-I-NA)
      DO 70 J=1,NC
70    MA(J,(NP+1-I))=MA(J,(NP+1-I-NA))
75    DO 71 I=1,NA
      DA(1+I+NPH+NPI)=0.0
      DO 71 J=1,NC
71    MA(J,(1+I+NPH+NPI))=0.0
72    IF(N5.NE.0) GO TO 74
      DO 73 I=1,NS
      DA(1+NPH+NPI+NA+I)=0.0
      DO 73 J=1,NC
73    MA(J,(1+NPH+NPI+NA+I))=0.0
74 .  NP=NI
      NORMP=0.0
      DO 31 I=1,NC
31    NORMP=NORMP+DPNU(I)**2
```

```
      NORMP=SQRT(NORMP)
      PRINT*," "
      PRINT 50,(PNU(I),I=1,NC)
      PRINT*," "
      PRINT 55,(DPNU(I),I=1,NC),NORMP
      PRINT*," "
      PRINT 56,(DA(I),I=1,NP)
      NORMA=0.0
      DO 32 I=1,NP
   32 NORMA=NORMA+DA(I)**2
      NORMA=SQRT(NORMA)
      DIF=(FNORMA-NORMA)/FNORMA
      FNORMA=NORMA
      PRINT*," "
      PRINT 57,NORMA
C
C  CONVERGENCE CRITERIA
C
      IF(NORMA.LT.0.3) GO TO 200
      IF(NORMA.LT.1.0E+3.AND.NORMA.GT.1.0E+2) P=P*1.0E-4
      IF(NORMA.LT.1.0E+2.AND.NORMA.GT.1.0E+1) P=P*1.0E-3
      IF(NORMA.LT.1.0E+1.AND.NORMA.GT.1.0E+0) P=P*1.0E-2
      IF(NORMA.LT.1.0E+0.AND.NORMA.GT.1.0E-1) P=P*1.0E-1
      GO TO 747
  200 IF(NORMM.LE.CC3.AND.NORMA.LE.CC4.AND.P.EQ.1.0) GO TO 201
      P=P*1.0E+2
      IF(P.GT.1.0) P=1.0
      GO TO 1000
  201 PRINT*," "
      PRINT*,"*CONVERGENCE*     PERFORMANCE INDEX= ",A(1),"      P= ",P
  757 STOP
      END
```

```
      SUBROUTINE GAUSD(M,EPS,B,C,KER,LAY)
      DIMENSION B(LAY,LAY),C(LAY,LAY),A(20,20),X(20,20)
      DOUBLE PRECISION Z,A,X,S,RATIO,EP
      DOUBLE PRECISION Z,A,X,S,RATIO,EP
      EP = EPS
      N = M
      DO 100  J = 1,N
      DO 100  K = 1,N
  100 A(J,K) = B(J,K)
       DO 1 I=1,N
       DO 1 J=1,N
    1 X(I,J) = 0.0
       DO 2 K=1,N
    2 X(K,K) = 1.0
   10  DO 34 L=1,N
       KP=0
      Z = 0.0
       DO 12 K=L,N
      IF(Z-DABS(A(K,L))) 11,12,12
   11 Z = DABS(A(K,L))
      KP=K
   12  CONTINUE
       IF(L-KP)13,20,20
   13  DO 14 J=L,N
       Z=A(L,J)
       A(L,J)=A(KP,J)
   14  A(KP,J)=Z
       DO 15 J=1,N
       Z=X(L,J)
       X(L,J)=X(KP,J)
   15  X(KP,J)=Z
   20 IF(DABS(A(L,L))-EP)50,50,30
   30  IF(L-N)31,34,34
   31  LP1=L+1
       DO 36 K=LP1,N
```

```
      IF(A(K,L))32,36,32
   32 RATIO=A(K,L)/A(L,L)
      DO 33 J=LP1,N
   33 A(K,J)=A(K,J)-RATIO*A(L,J)
      DO 35 J=1,N
   35 X(K,J)=X(K,J)-RATIO*X(L,J)
   36 CONTINUE
   34 CONTINUE
   40 DO 43 I=1,N
      II=N+1-I
      DO 43 J=1,N
     S = 0.0
      IF(II-N)41,43,43
   41 IIP1=II+1
      DO 42 K=IIP1,N
   42 S=S+A(II,K)*X(K,J)
   43 X(II,J)=(X(II,J)-S)/A(II,II)
      KER=1
     DO 200  J = 1,N
     DO 200  K = 1,N
  200 C(J,K) = X(J,K)
      GO TO 75
   50  KER=2
   70 PRINT 71
   71 FORMAT(1X,*MATRIX SINGULAR IN GAUSD*)
   75  CONTINUE
     RETURN
       END
```

```
      SUBROUTINE RK45(T,X)
C     FIFTH-ORDER RUNGE-KUTTA INTEGRATION WITH STEPSIZE CONTROL
C     FEHLBERG COEFFICIENTS
      DIMENSION ALPH(6),B(6,5),CH(6),CD(6),AA(22),TOT(5)
      DIMENSION X(6),XDUM(6),F1(6),F2(6),F3(6),F4(6),F5(6),F6(6)
      COMMON/MISC/NPH,NPI,NA,NS,TF,DT,N,S,LL,NN,PP,QQ,AA,TOT
      DATA (ALPH(I),I=1,6)/
     10000000000000000000B,17164000000000000000B,17166000000000000000B
     21717730473047304730 58,17204000000000000000B,17174000000000000000B
      DATA (B(I),I=1,30)/
     10000000000000000000B,17164000000000000000B,17146000000000000000B
     11717702174344304541 2B,17214041136641136640B,60613204573204573204B
     12*00000000000000000000B,17164400000000000000B,60561344115275735026B
     16054377777777777777B,17214000000000000000B,3*00000000000000000000B
     11721651045771004333 0B,17227130647130647130B,60572362247213732362B
     14*00000000000000000000B,60521322455322455322B,17167176601176601176B
     15*00000000000000000000B,60613463146314631463B/
      DATA (CH(I),I=1,6)/
     11714745347262200745 4B,00000000000000000000B,17174115611213152432B
     21717403107252547724 4B,60622172702436560507B,17134517101123620225B
      DATA (CD(I),I=1,6)/
     16070223722372237223 6B,00000000000000000000B,17127524375331472673B
     21712736322354277657 7B,60652702436560507533B,60643260676654157552B
      DATA ERPS,TOL,TOLT/1.E-12,1.E-09,1.E-10/
      IP=1  $  IS=0  $  IR=0
      IP=5
    9 IF(ABS(DT).GE.ABS(TF-T)) DT = TF-T
   10 DO 20 I=1,N
   20 XDUM(I) = X(I)
      CV=SQRT(311302.98/(1.0-.0000069*X(3))**4.256)
      DEL1=CV-X(4)
   12 TS=T+ALPH(1)*DT
      CALL DERIV(TS,X,F1)
      IF(MOD(IS,IP).EQ.0.AND.S.EQ.1.0) PRINT 4,T,X
    4 FORMAT(1X,7E15.7)
```

```
   30 CONTINUE
      B21 = B(2,1)*DT
      TS = T + ALPH(2)*DT
      DO 42 I=1,N
   42 X(I) = B21*F1(I) + XDUM(I)
      CALL DERIV(TS,X,F2)
      B31 = B(3,1)*DT $ B32 = B(3,2)*DT
      TS = T + ALPH(3)*DT
      DO 43 I=1,N
   43 X(I) = B31*F1(I) + B32*F2(I) + XDUM(I)
      CALL DERIV(TS,X,F3)
      B41 = B(4,1)*DT $ B42 = B(4,2)*DT $ B43 = B(4,3)*DT
      TS = T + ALPH(4)*DT
      DO 44 I=1,N
   44 X(I) = B41*F1(I) + B42*F2(I) + B43*F3(I) + XDUM(I)
      CALL DERIV(TS,X,F4)
      B51 = B(5,1)*DT $ B52 = B(5,2)*DT $ B53 = B(5,3)*DT
      B54 = B(5,4)*DT
      TS = T + ALPH(5)*DT
      DO 45 I=1,N
   45 X(I) = B51*F1(I) + B52*F2(I) + B53*F3(I) + B54*F4(I) + XDUM(I)
      CALL DERIV(TS,X,F5)
      B61 = B(6,1)*DT $ B62 = B(6,2)*DT $ B63 = B(6,3)*DT
      B64 = B(6,4)*DT $ B65 = B(6,5)*DT
      TS = T + ALPH(6)*DT
      DO 46 I=1,N
   46 X(I) = B61*F1(I) + B62*F2(I) + B63*F3(I) + B64*F4(I) + B65*F5(I)
     1     + XDUM(I)
      CALL DERIV(TS,X,F6)
      C1 = CH(1)*DT  $  C3 = CH(3)*DT
      C4 = CH(4)*DT  $  C5 = CH(5)*DT  $  C6 = CH(6)*DT
      DO 100 I=1,N
  100 X(I) = XDUM(I) + C1*F1(I) + C3*F3(I) + C4*F4(I)
     1     + C5*F5(I) + C6*F6(I)
      CD1 = CD(1)*DT  $  CD3 = CD(3)*DT
```

```
      CD4 = CD(4)*DT  $  CD5 = CD(5)*DT  $  CD6 = CD(6)*DT
C     ESTIMATE NEW STEP SIZE
      ER = 0.0
      DO 150 J=1,N
      A = X(J)
      IF(ABS(A).LT.ERPS) GO TO 150
      ES = ABS((CD1*F1(J) + CD3*F3(J) + CD4*F4(J) +
     1     CD5*F5(J) + CD6*F6(J))/A)
      IF(ES.GT.ER) ER = ES
  150 CONTINUE
      ER = ER + 1.0E-20
      DT1 = DT
      DT = TOLT/ER
      DT = DT1*DT**.2
      IF(ER-TOL) 7,7,8
    8 IR = IR + 1
      GO TO 30
    7 CONTINUE
C TOLERANCE IS PASSED
      TDUM=T+DT1
      CV=SQRT(311302.98/(1.0-.0000069*X(3))**4.256)
      DEL2=CV-X(4)
      SS1=SIGN(1.0,DEL1)
      SS2=SIGN(1.0,DEL2)
      IF(ABS(DEL1).LT.1.0E-8.AND.SS1.NE.SS2) GO TO 999
      IF(SS1.EQ.SS2) GO TO 180
      DT=ABS((DEL1*DT1)/(DEL2-DEL1))
      IF(ABS(DEL2).GE.1.0E-8) GO TO 30
  999 DT=DT1
      IF(LL.EQ.0) GO TO 1
      GO TO 2
    1 LL=1
      GO TO 3
    2 LL=0
    3 IF(S.EQ.1.0) PRINT*,"LL= ",LL
      IF(S.EQ.1.0) PRINT 4,T,X
```

```
  180 CONTINUE
      T = T + DT1
      IF(TF-T) 5,5,6
    6 IS = IS + 1
      GO TO 9
    5 IF(S.EQ.1.0) PRINT 4,T,X
      RETURN $ END
```

```
      SUBROUTINE DERIV(T,X,DX)
      DIMENSION X(6),DX(6),AA(22),TOT(5)
      COMMON/MISC/NPH,NPI,NA,NS,TF,DT,N,S,LL,NN,PP,QQ,AA,TOT
      COMMON/MISC1/BA,PI,SIG,ALF
      ROW=(1.0-0.0000068823*X(3))**4.2553
      CALL CALC(BA,PI,ALF,SIG,T,X,ROW)
      A=0.0000149166+0.00410241*ALF**2+.0000022377*ABS(SIG)
      B=0.0037295
      DX(1)=X(4)*COS(X(5))*COS(X(6))
      DX(2)=X(4)*COS(X(5))*SIN(X(6))
      DX(3)=X(4)*SIN(X(5))
      DX(4)=48.261*PI*COS(ALF)-A*ROW*X(4)**2-32.174*SIN(X(5))
      DX(6)=(48.261*PI*SIN(ALF)*SIN(BA)-16.087*SIG*COS(BA)+B*ROW*X(4)**2
     2*ALF*SIN(BA))/(X(4)*COS(X(5)))
      DX(5)=(48.261*PI*SIN(ALF)*COS(BA)+16.087*SIG*SIN(BA)+B*ROW*X(4)**2
     2*ALF*COS(BA)-32.174*COS(X(5)))/X(4)
      DO 1 I=1,6
    1 DX(I)=DX(I)*AA(1)
      RETURN
      END
```

```
      SUBROUTINE CALC(BA,PI,ALF,SIG,T,X,ROW)
      DIMENSION X(6),AA(22),TOT(5)
      COMMON/MISC/NPH,NPI,NA,NS,TF,DT,N,S,LL,NN,PP,QQ,AA,TOT
      CALL CHEBY(T,TOT)
      BA=0.0
      ALF=0.0
      SIG=0.0
      PI=0.0
      QQ=0.0
      PP=0.0
      NN=0
      DO 2 I=1,NPH
    2 BA=BA+AA(1+I)*TOT(I)
C     THRUST
      DO 3 I=1,NPI
    3 PI=PI+AA(1+I+NPH)*TOT(I)
      IF(PI.LT.0.0) PI=0.0
      IF(PI.GT.1.0) PI=1.0
      IF(PI.GE.1.0) NN=1
      IF(PI.LE.0.0) NN=-1
C     CLMAX LIMIT
      ALFLIM=0.2
      VC=558.1/(ROW**0.5)
      IF(X(4).LE.VC) GO TO 4
      ALFLIM=62286.8/(X(4)**2*ROW)
    4 CONTINUE
      DO 5 I=1,NA
    5 ALF=ALF+AA(1+I+NPH+NPI)*TOT(I)
      IF(ALF.GT.ALFLIM) ALF=ALFLIM
      IF(ALF.EQ.ALFLIM) PP=1.0
      IF(NS.EQ.0) GO TO 7
      DO 6 I=1,NS
    6 SIG=SIG+AA(1+I+NPH+NPI+NA)*TOT(I)
    7 IF(SIG.GT.2.0) SIG=2.0
      IF(SIG.LT.-2.0) SIG=-2.0
      IF(SIG.EQ.2.0) QQ=1.0
      IF(SIG.EQ.-2.0) QQ=-1.0
      RETURN
      END
```

```
      SUBROUTINE CHEBY(T,TOT)
      DIMENSION TOT(5),Z(4)
      Z(1)=T
      DO 1 I=2,4
      K=I-1
    1 Z(I)=Z(K)*Z(1)
      TOT(1)=1.0
      TOT(2)=2.0*Z(1)-1.0
      TOT(3)=8.0*Z(2)-8.0*Z(1)+1.0
      TOT(4)=32.0*Z(3)-48.0*Z(2)+18.0*Z(1)-1.0
      TOT(5)=128.0*Z(4)-256.0*Z(3)+160.0*Z(2)-32.0*Z(1)+1.0
      RETURN
      END
```

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Abstract

The objective of the study is to find the optimal trajectories and corresponding minimum turning times for a high performance aircraft with and without direct sideforce to perform a prescribed turn. These trajectories and times are then compared to evaluate the benefit of direct sideforce. Optimal control theory is applied to solve the minimum time to turn optimal control problem using a suboptimal control problem approach and a second order parameter optimization method.

The results indicate that the use of direct sideforce is beneficial in reducing turning time. In addition, the use of sideforce causes a small loss of energy for initial velocities lower and much higher than the corner velocity.

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8